

Hydraulics & Laboratory

September 2012

Syllabus

Semester:	Fall 2012
Course No:	CIVE 440
Course Title:	Hydraulics & Laboratory
Prerequisites:	CIVE 340 Fluid Mechanics
Class Schedule:	Tu. & Th. 12:30–13:50; Tu. & Th. 14:00–15:20 or On-Call-Basis
Classroom:	Bechtel Rm. 545
Instructor:	Habib Basha
Office:	Bechtel Rm. 535
Hours:	Tu. & Th.: 11:30–12:30 & 15:30–16:30 or by appointment

Topics

Flow in Conduits; Flow in Open-Channels; Flow Measurements; Laboratory Experiments.

Textbook

Crowe, C. T., Elger, D. F., and Roberson, J. A., *Engineering Fluid Mechanics*, 9th ed., chap. 10, 13, & 15, John Wiley & Sons, New York, 2010.

Course Description

Hydraulics deals with the application of fluid mechanics in the analysis and design of hydraulic structures and water resources systems. The course is important in that it provides the technical background in the design of water resources projects. The primary applications are in the analysis of flow in water distribution systems and the design of canals and related structures for irrigation projects. Another important application is in measurements of discharge in conduits and in channels. The course is divided in two main parts covering pipe flow and open-channel flow. Laboratory experiments dealing with flow measurements and pipe friction are also conducted.

Course Learning Outcomes

- Students will know the fundamental hydraulic concepts.
- Students will have the ability to solve pipe-flow problems.
- Students will have the ability to compute water-surface profiles in open-channels.
- Students will have the ability to design basic hydraulic structures.
- Students will have the ability to measure discharge in water systems.
- Students will have had a hands-on experience of hydraulics in the laboratory.
- Students will be skilled in the use of major technical software.
- Students will have basic technical writing skills.

Course Outline

1. Introduction
 - a. Hydraulics Projects
 - b. Hydraulics Course Preview
 - c. Review of Fluid Mechanics
2. Steady Pipe Flow
 - a. Energy Equation
 - b. Head Loss Equations
 - c. Single Pipe Flows
 - d. Pump Curves, EGL
 - e. Parallel & Branching Pipes
 - f. Flow in Pipe Manifolds
 - g. Flow in Pipe Networks
3. Flow in Open-Channels
 - a. Uniform Flow & Manning's Equation
 - b. Critical Flow: Specific Energy & Critical Depth
 - c. Channel Flow Measurement: Weirs
 - d. Spillways and Hydraulic Jump
 - e. Gradually Varied Flow & Water Surface Profiles
 - f. Direct & Standard Step Method
 - g. Overbank Flow & Flood Plain Encroachment
 - h. Lake Discharge & Short Channels
 - i. Design of Channels & Best Hydraulic Section
 - j. Design of Channel Transitions
 - k. Design of a Stilling Basin
4. Laboratory Experiments
 - a. Pipe Friction
 - b. Weirs
 - c. Spillway & Hydraulic Jump

Course Assessment

Exam I	(5 Problems)	30%
Exam II	(5 Problems)	40%
Laboratory Reports	(1 Group + 1 Individual report)	20%
Project: EPANET Software Application	(Group report)	10%
Participation		±5%

Deadlines and Timelines

Exams:	Exam I 01-Nov-12	Exam II 13-Dec-12	
Project:	Selection 13-Nov-12	Report 09-Jan-13	Presentation 10-Jan-13

Course Policy

The final grade is based on a weighted average of the above components. The above weights might be modified to the best interest of the class. Class participation is rewarded in the final grade as shown above. There are no grade push-ups and no make-up exams. Project assignment is due as scheduled. Late submission will be penalized at 5 percentage points per day. Plagiarism is a serious offence and will be treated as such.

Course Calendar

2012	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Week	Topics	Timelines-Deadlines
SEP	16	17	18	19	20	21	22	1	Introduction – Syllabus	
	23	24	25	26	27	28	29	2	Pipe flows theory	
	30	1	2	3	4	5	6	3	Pipe flows application	
	7	8	9	10	11	12	13	4	Branching reservoirs	
OCT	14	15	16	17	18	19	20	5	Manifolds & networks	Lab Experiment 1
	21	22	23	24	25	26	27	6	Uniform flow	
	28	29	30	31	1	2	3	7	Uniform flow	
NOV	4	5	6	7	8	9	10	8	Critical flow – Weirs	Lab Experiment 2
	11	12	13	14	15	16	17	9	Spillway	
	18	19	20	21	22	23	24	10	Hydraulic jump	
	25	26	27	28	29	30	1	11	Water surface profiles	Lab Experiment 3
DEC	2	3	4	5	6	7	8	12	Water surface profiles	
	9	10	11	12	13	14	15	13	Transition structures	
	16	17	18	19	20	21	22	14	Final exam problems	
	23	24	25	26	27	28	29		Review problems	
	30	31	1	2	3	4	5			Lab Report Due
JAN	6	7	8	9	10	11	12			Project Report Due
	13	14	15	16	17	18	19			Exam Dates

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1. Introduction

Hydraulic engineering deals with the analysis and design of hydraulic structures and water resources systems. The primary applications are in the analysis of flow in water distribution systems and the design of canals and related structures for irrigation projects. Another important application is in measurements of discharge in conduits and in channels.

1.1 Water Networks

Water distribution network analysis is an important problem in civil engineering. Water networks have three major components: pumping stations, distribution lines, and storage facilities.

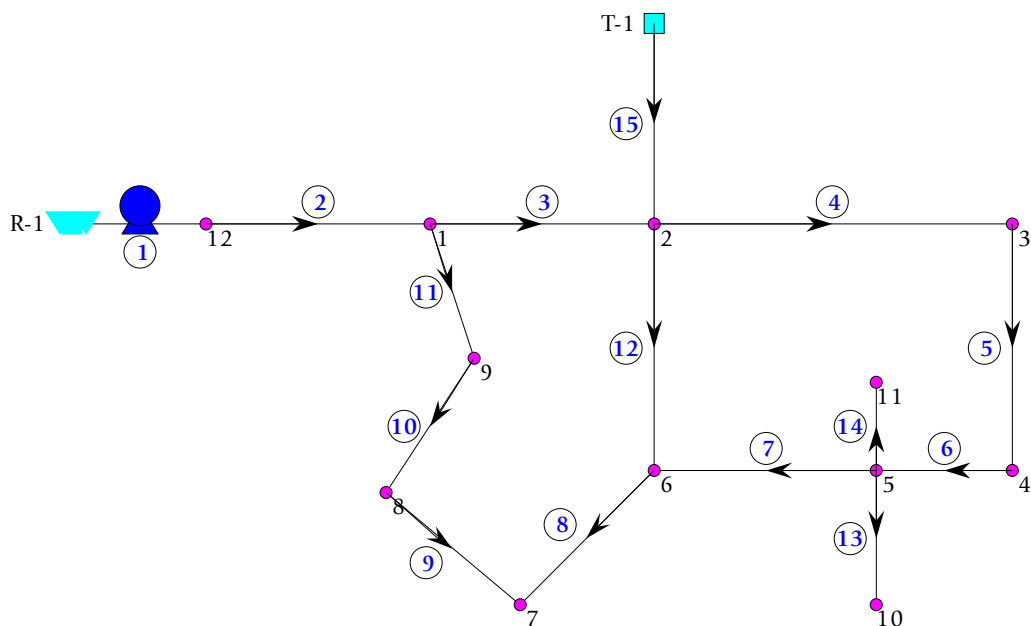


Figure 1-1 A typical water distribution network

Water distribution systems have one or more sources of potable water from groundwater or surface water and numerous loads or withdrawals for residential, commercial, agricultural and industrial establishments. The system must be able to supply the demand of water needed with adequate pressure for a number of different loading conditions.

Pipes are the most abundant element in the network. The primary lines carry flow from the pumping station to and from elevated storage tanks. The secondary lines form smaller loops and run from one primary line to another. Pipe sections may contain valves to regulate the flow or pressure in the system.

Nodes refer to the end section of a pipe. There are two categories of nodes: junction nodes where the demand or withdrawal is known and fixed-grade nodes such as a reservoir connection where the pressure or the head is known. Storage facilities are needed to meet the fluctuations in demand and to stabilize the pressures in the system and dampen out hydraulic transients. Water is pumped into a reservoir when the demand is low and withdrawn by gravity flow during periods when the demand is high.

Introduction

1.2 Irrigation System

A second major water resource system is an irrigation project whereby water is diverted from a river to irrigate agricultural lands. A diversion dam is built so as to maintain a high water level behind it that will direct the water into the intake structure and the irrigation canal. The transition structure is useful to take the water across a depression or small valley without causing significant head losses. The purpose of the check structures is to control the flow through gates and divert water into secondary canals. In case of a significant drop in the land surface, drop structures along with stilling basins are necessary to control the flow and reduce the kinetic energy and possible erosion of the downstream soil.

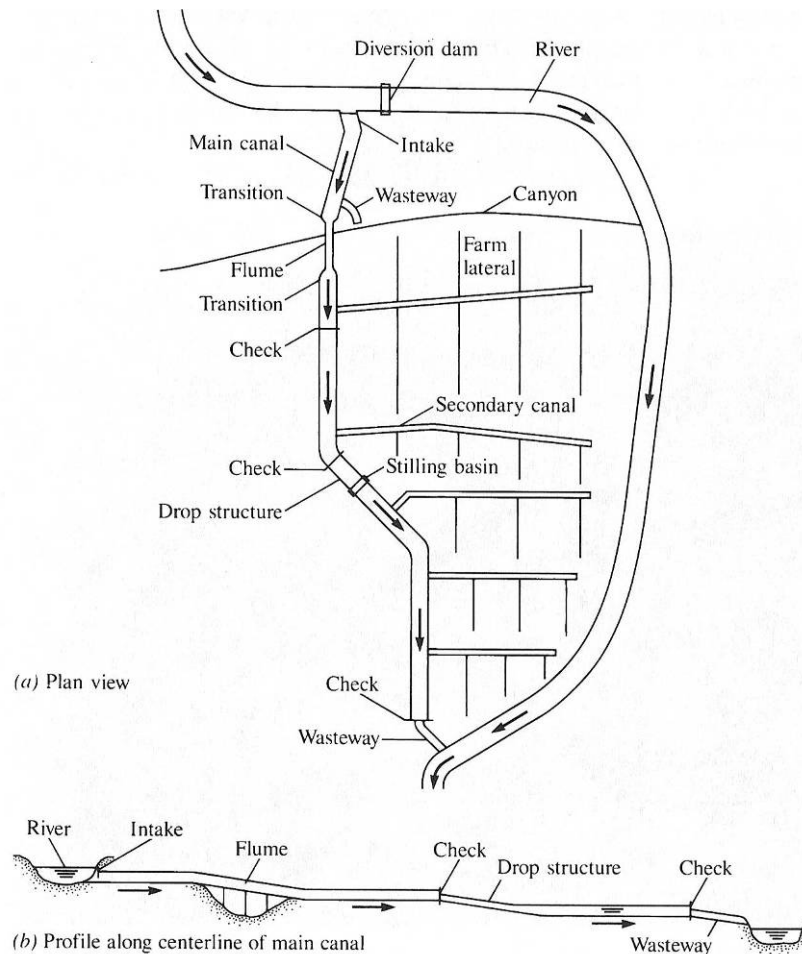


Figure 4-8 Layout of irrigation system (a) Plan view (b) Profile along centerline of main canal

Figure 1-2 A typical irrigation system (Source: *Roberson et al.* [1998])

2. Steady Pipe Flows

Flow in conduits arises in many fields of engineering. Water distribution systems are one prime application of the theory of conduit flows. The basic principles governing closed conduit flows are: conservation of mass, conservation of energy, and conservation of momentum. The continuity and energy equations are used to design a pipe system, while the momentum equation is employed to determine the forces acting on bends for a given discharge.

2.1 Continuity Equation

The continuity equation is expressed by

$$Q_{\text{in}} - Q_{\text{out}} = \frac{dS}{dt} \quad (2.1)$$

where Q_{in} and Q_{out} are the flow rates entering and exiting the system, and S is the volume of water stored in the system. For steady flow, eq. (2.1) simplifies to $Q_{\text{in}} = Q_{\text{out}}$ or

$$A_{\text{in}} V_{\text{in}} = A_{\text{out}} V_{\text{out}} \quad (2.2)$$

where A is the cross-sectional area of the flow.

2.2 Energy Equation

The energy equation is given by

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \quad (2.3)$$

where p/γ is the pressure head (i.e. the energy possessed by the liquid in virtue of its pressure), $V^2/2g$ is the velocity head (i.e. the kinetic energy per unit weight of a flowing liquid), z is the elevation head (potential energy), h_p is the head supplied by the pump, h_t is the head supplied by the turbine, and h_L is the head loss between section 1 and 2.

The correction factor α in the velocity head term accounts for the assumption of a mean velocity in (2.3) whereas the actual velocity distribution is varying from a maximum at the centerline to zero at the boundaries

$$\alpha = \frac{1}{AV^3} \int u^3 dA \quad (2.4)$$

Here u is the actual velocity distribution and V is the mean velocity. The parameter α ranges from 1.01 to 1.15 and is usually assumed to be unity in pipe flow computations.

The sum of the pressure head, the velocity head, and the elevation head is defined as the hydraulic head H

$$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z \quad (2.5)$$

Water flows from a higher head H to a lower head. Eq. (2.3) can then be expressed as $H_1 + h_p = H_2 + h_t + h_L$. It applies to incompressible steady flow of a real viscous fluid flowing from an inlet section to an outlet section. If $h_L = 0$, the fluid is assumed to be inviscid, i.e. frictionless. For $h_p = h_t = h_L = 0$, eq. (2.3) reduces to the Bernoulli equation.

Steady Pipe Flows

2.3 Laminar and Turbulent Flow

Flow of water is mostly turbulent; the flow of oil can be laminar. The flow is considered laminar when the Reynolds number is less than 2000, and it is turbulent for Reynolds number greater than 3000. The velocity profile in laminar flows is parabolic while the velocity distribution in turbulent flows is of the form of a power law formula.

2.4 Head Loss Equations

2.4.1 Pipe Friction Losses

In laminar flows, the head loss is proportional to V . The mathematical equations become easier to solve since they are linear. On the other hand, the head loss in turbulent flows is proportional to V^n . The governing equations are therefore nonlinear and more difficult to solve. Dimensional analysis and experimental investigations are used to come up with the head loss equation.

For laminar flow in pipes, the friction equation can be derived from the shear stress distribution and the parabolic velocity distribution. It is expressed by

$$h_f = \frac{32\mu LV}{\gamma D^2} \quad (2.6)$$

For turbulent flows, the head loss in a pipe is given by

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \quad (2.7)$$

The friction factor f can be found from the Moody diagram or from the following equation valid for $10^{-5} < k_s/D < 2 \cdot 10^{-2}$ and for $4000 < R_e < 10^8$

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s}{3.7D} + \frac{2.51}{R_e \sqrt{f}} \right) \quad (2.8)$$

An alternate equation, which expresses the friction factor f in an explicit fashion, is given by

$$f = \frac{0.25}{\left[\log_{10} \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^2} \quad (2.9)$$

For noncircular pipes, the friction equation (2.7) becomes

$$h_L = \frac{f}{4} \frac{L}{R_h} \frac{V^2}{2g} \quad (2.10)$$

where the hydraulic radius R_h is defined as

$$R_h = \frac{A}{P} \quad (2.11)$$

Here A is the cross-sectional area of the flow and P is the wetted perimeter. For a circular pipe flowing full, $R_h = D/4$. The hydraulic radius concept is substituted for D in non-circular cross sections and is used extensively in open channel flows and free surface flows.

2.4.2 Pipe Fitting Losses

Besides the friction loss in the pipe, there are also minor losses in pipe fittings, bends, valves, entrances and exits. All minor losses are expressed in terms of

$$h_m = k \frac{V^2}{2g} \quad (2.12)$$

where k is the loss coefficient (for entrance losses, sudden contraction and expansion, pipe fittings) given in figures and tables in textbooks. For submerged exit loss, $k = 1$.

2.5 Power, EGL and HGL

The power in Watts is expressed as

$$P = \gamma QH \quad (2.13)$$

where H is the head. The power of a jet is for $H = V^2/2g$, the power lost in fluid friction is for $H = h_L$, and the power of a machine is for $H = h_m$.

The hydraulic grade line (HGL) is the sum of the pressure and elevation head $p/\gamma + z$, and the energy grade line (EGL) is the plot of the hydraulic head H (2.5). The EGL decreases at a rate of head loss. The HGL and EGL are useful for spotting regions of low pressures without the need for calculating the pressure at fine discrete intervals. If the hydraulic grade line between two points fall below the pipe system, then that region has a low pressure.

2.6 Solution of Pipe-Flow Problems

There are basically three types of problems:

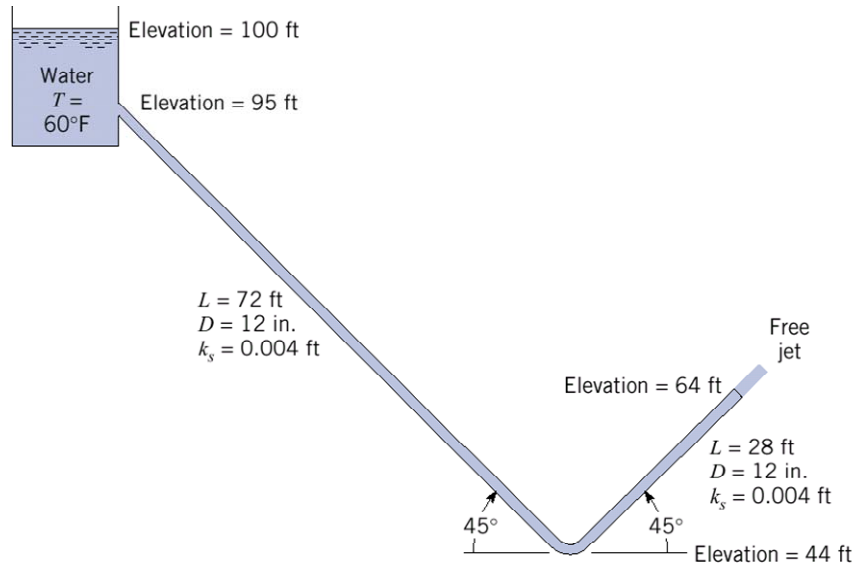
1. Determine the head loss, given the flow rate, kind and size of pipe.
2. Determine the flow rate, given the head, kind and size of pipe
3. Determine the size of pipe, given the head, flow rate, and kind of pipe.

The solution of pipe flow problems consists of writing the energy equation between any two points for which the pressure and/or velocity are known. If there are two unknowns, one must use the continuity equation also. The solution of some problems might involve a trial and error procedure to find the velocity or flow rate. Remember that all relevant loss terms must be included.

Steady Pipe Flows

Example 2.1: Minimum Pressure

Calculate the discharge in the pipe and determine the points of maximum and minimum pressure in the system. Assume $K_b = 0.2$, $\nu = 1.22 \times 10^{-5} \text{ ft}^2/\text{s}$ and $\gamma = 62.4 \text{ lb/ft}^3$.



Solution

Expressing the energy equation between point 1 at the reservoir and point 2 at the free jet, one obtains

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_t = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(\sum f \frac{L}{D} + \sum k_m \right) \frac{V_2^2}{2g}$$

Therefore,

$$100 = 64 + \left(1 + 0.5 + f \frac{100}{1} + 0.2 \right) \frac{V_1^2}{2g}$$

Assuming $f = 0.028$, one obtains $V = 22.7 \text{ ft/s}$ and $R_e = 1.9 \times 10^6$ and the new value of f is $f = 0.028$. No further iteration is needed. The flow rate is then $Q = 17.58 \text{ ft}^3/\text{s}$.

The point of minimum pressure is at the entrance of the pipe or at the exit, i.e. zero. The pressure at the entrance is

$$100 = \frac{P_e}{\gamma} + 95 + (1 + 0.5) \frac{(22.7)^2}{2g}$$

Hence, $P_e = -7\gamma$, i.e. $P_e = -3 \text{ psig}$ is the minimum pressure.

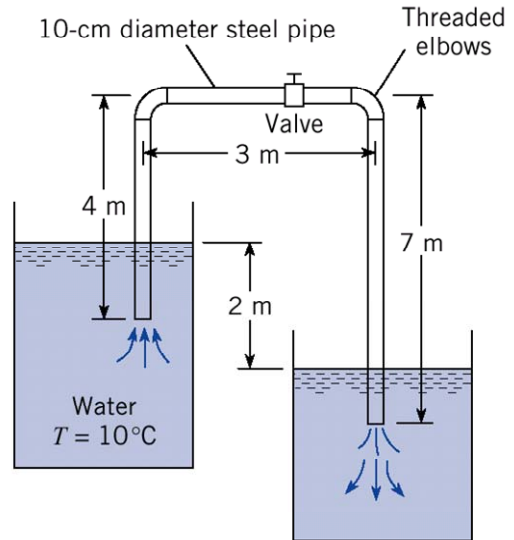
The maximum pressure is just before the bend

$$100 = \frac{P_m}{\gamma} + 44 + \left(1 + 0.5 + 0.028 \frac{72}{1} \right) \frac{(22.7)^2}{2g}$$

Hence, $P_m = 27.9 (62.4) = 1740 \text{ lb/ft}^2$ is the maximum pressure.

Example 2.2: Loss Coefficient

Compute the loss coefficient K_v of the partially-closed valve that reduces the discharge to 50% of the flow with the valve wide open. The equivalent sand-grain roughness of steel k_s is 0.046 mm and the kinematic viscosity is $1.31 \times 10^{-6} \text{ m}^2/\text{s}$



Solution

Expressing the energy equation between the two reservoirs, one obtain

$$2 = \left[0.5 + 2(0.9) + .2 + 1 + f \frac{14}{0.1} \right] \frac{V^2}{2g}$$

Assuming $f = 0.018$, one gets $V = 2.55 \text{ m/s}$. Iterating again, one obtains $f = 0.0188$ and the velocity becomes $V = 2.53 \text{ m/s}$. The flow rate is then $Q = 0.02 \text{ m}^3/\text{s}$.

For one-half the discharge, $Q = 0.01 \text{ m}^3/\text{s}$, the corresponding velocity is $V = 1.27 \text{ m/s}$. The corresponding $f = 0.02$. Hence

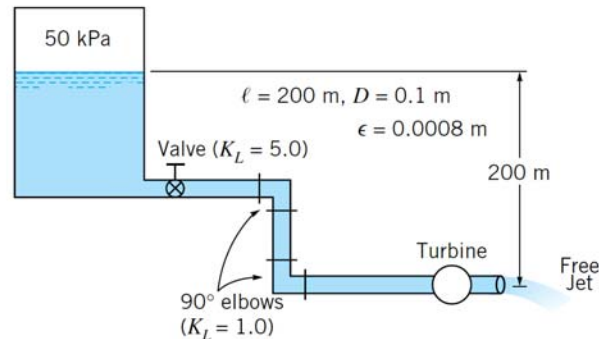
$$2 = \left[0.5 + 2(0.9) + k_v + 1 + .02 \frac{14}{0.1} \right] \frac{1.27^2}{2g}$$

Solving for K_v , one gets $K_v = 18.2$.

Steady Pipe Flows

Example 2.3: Turbine Problem

Water drains from a pressurized tank through a pipe system as shown in the Figure. Compute the flow rate if the head of the turbine is equal to 116 m. The length of the pipe is 200 m. The fluid properties are $\gamma = 9.8 \text{ kN/m}^3$ and $\nu = 1.12 \times 10^{-6} \text{ m}^2/\text{s}$.



Solution

Expressing the energy equation between point 1 inside the tank and point 2 at the free jet, one obtains

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_t = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(\sum f \frac{L}{D} + \sum k_m \right) \frac{V_2^2}{2g}$$

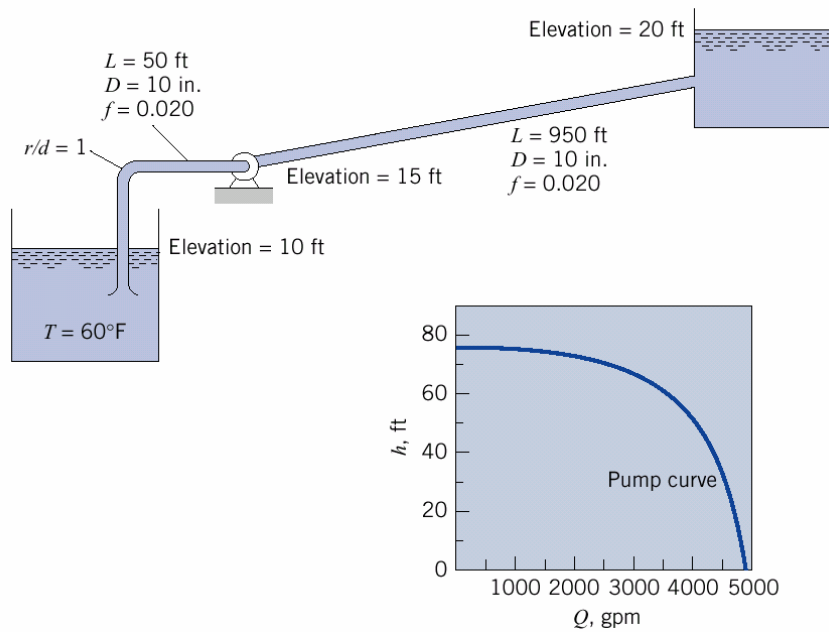
Therefore,

$$\frac{50}{9.81} + 200 - 116 = \left(1 + f \frac{200}{0.1} + 0.5 + 5 + 2 \right) \frac{V_1^2}{2g}$$

Here $V_1 = V_2 = (4/\pi)Q/0.1^2$. Assuming $f = 0.03$, one obtains $V = 5.05 \text{ m/s}$. The Reynolds number is then $R_e = 4.5 \times 10^5$ and the new value of f is $f = 0.0354$. Further iteration yields $V = 4.69 \text{ m/s}$ and $f = 0.0355$. The flow rate is then $Q = VA = 0.0368 \text{ m}^3/\text{s}$.

Example 2.4: Pump Curve

Calculate the discharge of water in the following system



Solution

Expressing the energy equation between the lower reservoir and the upper reservoir, one obtains

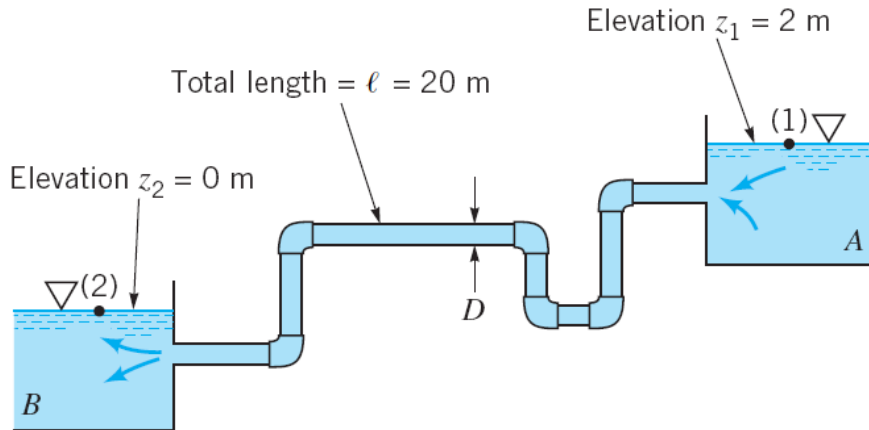
$$0 + 0 + 10 + h_p = 0 + 0 + 20 + \left(0.1 + 0.020 \frac{1000}{10/12} + 1 \right) \frac{V_1^2}{2g}$$

Now $V_1 = Q/A = Q/0.545$. Plotting this equation on the pump curve yields $Q = 2950$ gpm.

Steady Pipe Flows

Example 2.5: Pipe Sizing

Determine the pipe diameter needed for water ($\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$) to flow from reservoir *A* to reservoir *B* through a cast-iron pipe ($k_s = 0.25 \text{ mm}$) of length 20 m at a rate of $Q = 0.002 \text{ m}^3/\text{s}$. The system contains a sharp-edged entrance ($K = 0.5$) and six threaded 90° elbows ($K = 1.5$).



Solution

Expressing the energy equation between reservoir *A* and reservoir *B*, one obtains

$$0 + 0 + 2 = 0 + 0 + 0 + \left(f \frac{20}{D} + 0.5 + 6(1.5) + 1 \right) \frac{V^2}{2g}$$

The solution proceeds by trial and error. One assumes D , solves for V using the continuity equation and determines f from the Moody's diagram or

$$f = \frac{0.25}{\left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2}$$

The results for various trial runs are tabulated below

D (cm)	A (m^2)	V (m/s)	Re	f	E_B (m)
2	3.14	6.37	111690	0.0416	101.5
5	0.002	1.0186	44675	0.0327	1.089
4.5	0.0016	1.2575	49639	0.0334	1.802
4.4	0.0015	1.3153	50767	0.0336	2.007

2.7 Conduit Systems

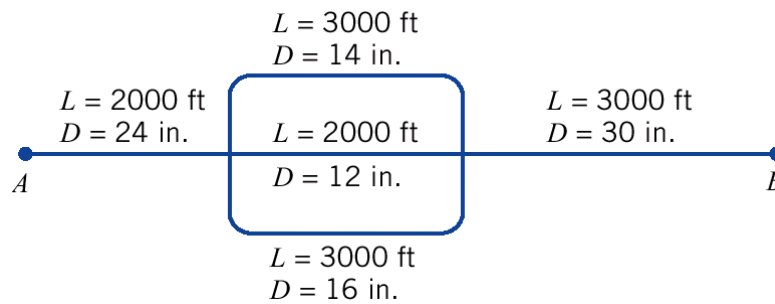
Flow in a series of pipes is the simplest pipe system. Other applications include flow in parallel pipes, flow in branching pipes, flow in a pipe manifold, and flow in pipe networks.

For flow in pipes in parallel, the head loss is the same. The problem is to determine the division of flow in each pipe, given the total flow rate. Similarly, the problem of branching pipes is to determine the discharge in each pipe that is connected to a reservoir with a given head.

Manifolds are pipes that branch into other pipes. They can be of the combining flow type or the dividing flow type. An example of a dividing flow type is a diffuser for disposal of sewage. The distribution of the flow can also be accomplished by ports cut in the manifold. The discharge through each port is different as the head in the pipe changes along the length of the manifold because of friction and momentum change.

Problem 2.1: Parallel Pipes

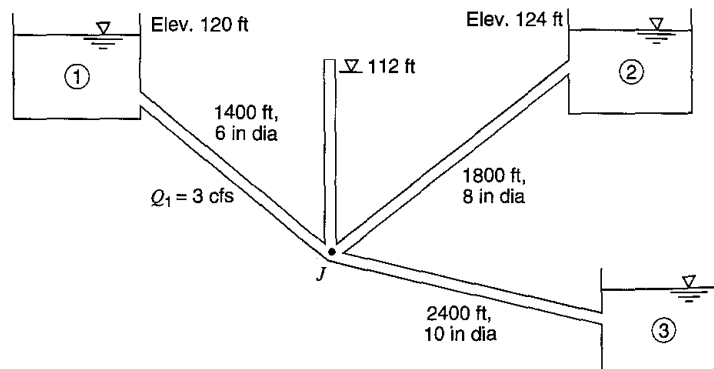
The pipes shown in the system are all concrete. Compute the head loss from A to B and the flow rates in the pipe system given that the total flow rate is 25 cfs.



Steady Pipe Flows

Example 2.6: Branching Pipes

Compute the flow rates and the elevation of the third reservoir given that the pipes are galvanized iron with $k_s = 0.0005$ ft and $\nu = 1.06 \times 10^{-5}$ ft²/s.



N.B.: the flow rate value shown in pipe 1 is incorrect.

Solution

The piezometric head at point J is 112 ft.

Expressing the energy equation between reservoir 1 and point J , one obtains for $f = 0.020$

$$120 = 112 + \left(1 + 0.020 \frac{1400}{0.5} \right) \frac{V_1^2}{2g}$$

Hence $V_1 = 3$ ft/s and $Q_1 = 0.6$ cfs.

Expressing the energy equation between reservoir 2 and point J , one obtains for $f = 0.020$

$$124 = 112 + \left(1 + 0.020 \frac{1800}{8/12} \right) \frac{V_2^2}{2g}$$

Hence $V_2 = 3.7$ ft/s and $Q_2 = 1.3$ cfs.

From continuity, the flow rate in pipe 3 is 1.9 cfs and the velocity is then 3.5 ft/s.

Expressing the energy equation between point J and reservoir 3, one obtains for $f = 0.020$

$$z_3 = 112 + \left(1 - 0.020 \frac{2400}{10/12} \right) \frac{(3.5)^2}{2g} = 101.2 \text{ ft}$$

The above values can be further corrected by updating the values of the friction coefficient using the Moody's diagram: $f_2 = 0.019$ and $f_3 = 0.018$.

Example 2.7: Pipe Manifold System

Compute the flow rates in a pipe manifold system and determine the water surface elevation in the upstream reservoir if the discharge rate at the last port in the dead end pipe is 2 cfs. The diameter of the manifold pipe is 12" and the diameter of the ports is 4". There are 5 discharge openings that are 10' spaced apart. The manifold is discharging into a pond at a depth of 10' and the distance from the reservoir to the first opening is 500'. Assume orifice flow in the ports with $C_d = 0.675$ and use a constant $f = 0.02$.

Solution

The head at each port is obtained from the energy equation

$$H_i = H_{i+1} + f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}$$

where

$$f = \frac{1}{4} \left[\log_{10} \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^2$$

The flow discharge through each port is given by

$$q_i = K_i a_i \sqrt{2g(H_i - H_0)}$$

where the flow coefficient K is

$$K_i = 0.675 \sqrt{1 - \frac{V_{i+1}^2}{2g(H_i - H_0)}}$$

The flow rates in the manifold system is obtained from the continuity equation at each port

$$Q_i = Q_{i+1} + q_i$$

The solution in tabular form is as follows:

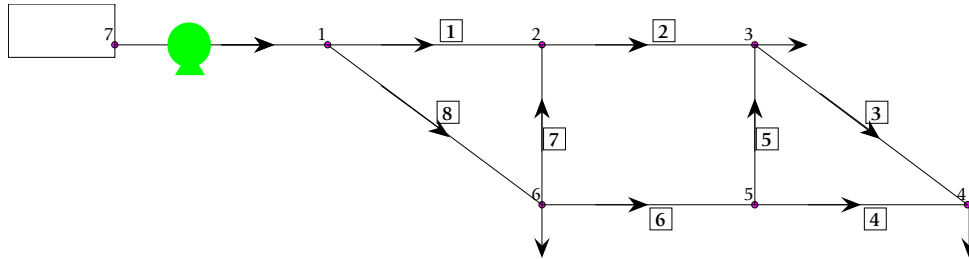
Port or Segment	q (cfs)	Q (cfs)	V (fps)	K	Re 10^3	f	h_1 (ft)	H (ft)
5	2			0.6750				27.9008
4	1.9955	2	2.5465	0.6731	181.9	0.02	0.0201	27.9209
3	1.9831	3.9955	5.0872	0.6674	363.4	0.02	0.0804	28.0013
2	1.9651	5.9786	7.6122	0.6581	543.7	0.02	0.1800	28.1812
1	1.9439	7.9437	10.1142	0.6454	722.4	0.02	0.3177	28.4989
Rsv		9.8876	12.5893		899.2	0.02	24.6102	53.1091

Therefore, the water surface elevation in the reservoir should be 53 ft.

Steady Pipe Flows

Example 2.8: Network – 8 Pipes

Write down the system of equations for the given network and tabular data expressing all coefficients in numerical form. Determine also the head supplied by the pump.



The hydraulic data are:

Junction	1	2	3	4	5	6	7
Elevation (m)	250	260	265	265	270	280	220
Demand (l/min)	0	0	150	50	0	100	RSV

Pipe	1	2	3	4	5	6	7	8
Diameter (cm)	20	20	20	15	15	15	20	15
Length (m)	250	300	600	450	500	300	500	600

The pump curve is defined by the shutoff head of 200 m and the following curve points: (300 l/min, 180 m) and (600 l/min, 150 m). Use $f = 0.02$ for all pipes.

Solution

There are 9 unknown pipe flow rates. The system of equation is then made of 6 continuity equations at the junctions and 3 energy equations around the 3 loops.

The continuity equations are

$$\text{J-1: } Q_p - Q_1 - Q_8 = 0$$

$$\text{J-2: } Q_1 + Q_7 - Q_2 = 0$$

$$\text{J-3: } Q_2 + Q_5 - Q_3 = 0.15$$

$$\text{J-4: } Q_3 + Q_4 = 0.05$$

$$\text{J-5: } Q_6 - Q_5 - Q_4 = 0$$

$$\text{J-6: } Q_8 - Q_7 - Q_6 = 0.1$$

$$\text{Loop 126: } k_1 Q_1^x - k_7 Q_7^x - k_8 Q_8^x = 0$$

$$\text{Loop 2356: } k_2 Q_2^x - k_3 Q_3^x - k_6 Q_6^x + k_7 Q_7^x = 0$$

$$\text{Loop 345: } k_3 Q_3^x - k_4 Q_4^x + k_5 Q_5^x = 0$$

where $k = \frac{8fL}{g\pi^2 D^5}$

Pipe	1	2	3	4	5	6	7	8
k (min ² /m ⁵)	0.3586	0.4303	0.861	2.720	3.023	1.814	0.7172	3.627
k (s ² /m ⁵)	1291	1549	3099	9793	10881	6529	2582	13057

The pump must supply a flow of 300 l/min; hence, the head supplied is 180 m.

3. Flow in Open-Channels

The study of open-channel flows is essential to the computation of water surface profiles to define the levels of flood inundation. It is also important to the design of canals and related structures for irrigation projects.

Open-channel flows are classified as uniform and non-uniform flows. Uniform flows assume that the velocity along the streamline is constant, i.e. $\partial v/\partial x = 0$. Hence, the depth is uniform across the length of the channel. This situation may occur in channels of constant cross-section and slope (prismatic channels). In case of uniform flow, the flow depth is called the normal depth and is designated by y_n . The normal depth can be calculated using the Chezy equation or the more common Manning's equation.

Non-uniform flows are much more common than uniform flows. They are also two types of flow regimes in non-uniform flows: sub-critical flow (tranquil flow) and super-critical flow (rapid flow). In non-uniform flows, the flow is further classified into two categories based on the length over which the velocity and depth changes. For changes in y and V over short distances, the flow is classified as rapidly-varied flow, while for changes in y and V over a long stretch of the channel, the flow is referred to as gradually-varied flow. In rapidly-varied flow, the bottom friction is usually neglected because of the short distance, while in gradually-varied flows the friction is a significant component of the energy equation. Note that all the above flow classifications correspond to steady flows, i.e. $\partial v/\partial t = 0$. Unsteady uniform channel flow is beyond the scope of this chapter while the unsteady non-uniform flow is the most complicated open-channel flow and is reserved for advanced courses.

The primary engineering application of flow in open-channels is the calculation of the flow depth y along the channel or river. In uniform flows, the depth is constant and is calculated once. In non-uniform flows, the depth is varying with distance and is to be calculated at various space intervals Δx .

3.1 Uniform Flow

Uniform flow implies that the water surface, channel bottom and the energy grade line are all parallel. Uniform flow may be established over a long channel of constant cross-section such as man-made channels. There are two uniform flow equations: the Chezy formula (1775) and the Manning's equation (1890). They are both dependent on the cross-sectional area A , the wetted perimeter P , the hydraulic radius ($R = A/P$), the slope S , and a roughness coefficient C or n .

3.1.1 Manning's Equation

The Manning's equation is expressed by

$$Q = \frac{1}{n} AR^{2/3} S^{1/2} \quad (3.1)$$

Eq. (3.1) is normally used to compute the depth for a given flow rate. The calculation is iterative since the depth variable y appears in A and $R = A/P$ in a nonlinear fashion. The equation is explicit in y only in case of wide channels, i.e. $R \approx y$, and in triangular channels.

For the general trapezoidal channel, the equation for the area and the wetted perimeter are

$$A = by + my^2 \quad P = b + 2y\sqrt{1 + m^2} . \quad (3.2)$$

The area of a circular channel partially full is given by

$$A = \frac{\theta}{2\pi} \frac{\pi D^2}{4} + \frac{D}{2} \cos[(2\pi - \theta)/2] \frac{D}{2} \sin[(2\pi - \theta)/2] = \frac{D^2}{8} (\theta - \sin \theta) \quad (3.3)$$

Flow in Open-Channels

where θ is the lower arc angle. The wetted perimeter is $P = \theta D/2$. The hydraulic radius is

$$R_h = \frac{D}{4\theta}(\theta - \sin \theta) \quad (3.4)$$

The corresponding depth is

$$y = \frac{D}{2}[1 - \cos(\theta/2)] \quad (3.5)$$

3.1.2 Compound Channel

For a compound channel with a left and right overbank area, Manning's equation still applies by assuming that the flow is made up of separate additive parts, i.e. $Q = Q_l + Q_c + Q_r$, where each part is calculated with the corresponding area, perimeter, and roughness values.

3.1.3 Best Hydraulic Section

The optimum cross-section is defined as the section with the minimum friction, i.e. with the minimum perimeter or maximum hydraulic radius. For a given cross-sectional area A , the shape of the cross-section is obtained from $\partial Q/\partial y = 0$ or $\partial R/\partial y = 0$ where $R = A/P$, $A = y(b + my)$ and $P = b + 2y\sqrt{1 + m^2}$.

3.1.4 Flood Plain Encroachment

Flood plain encroachment is the narrowing of the channel overbank areas by construction of buildings or levees. The extent of the encroachment is calculated such that the resulting increase in the water level is not more than a maximum value (typically 1 ft). The computation is done using Manning's equation if the encroachment is over long enough distance so that uniform flow condition applies. Otherwise, the gradually varied flow equation is to be used for such calculations.

3.1.5 Design of Erodible Channels

For a channel to be constructed in erodible material, care must be taken that the velocity is not too large. Otherwise, the channel will erode. Two methods can be used for designing erodible channels: the permissible velocity method and the tractive force method. A side slope should be used such that it will be stable under all conditions. Tables can give approximate permissible side slope and velocities for different materials. Once the flow rate, the maximum velocity allowed, the roughness and the slope are known, the appropriate cross-section of the channel can be determined using Manning's formula.

3.2 Critical Flow

The specific energy is defined as the sum of the depth of flow and the velocity head

$$E = y + \frac{V^2}{2g} \quad (3.6)$$

Critical flow occurs when the specific energy is minimum for a given discharge. That is

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \frac{dA}{dy} = 0 \quad (3.7)$$

Eq. (3.7) can be expressed in terms of the top width T using $dA = Tdy$

$$\frac{Q^2 T}{g A^3} = 1 \quad (3.8)$$

The solution of (3.8) gives the critical depth for a given discharge and cross-section geometry. For flow depths less than critical, the flow is supercritical and for depths larger than the critical depth, the flow is subcritical. The large depth corresponds to slow moving flow (subcritical flow) while small depths correspond to fast moving flow (supercritical flow). The flow in canals and rivers is normally subcritical while the flow over spillways and in chutes is supercritical.

For a rectangular channel, $A = By$ and $T = B$, the critical depth is

$$y_c = \frac{V^2}{g} \quad (3.9)$$

Or

$$y_c = \sqrt[3]{\frac{Q^2}{g B^2}} = \sqrt[3]{\frac{q^2}{g}} \quad (3.10)$$

For critical flow conditions, the kinetic energy is $V^2/2g = y_c/2$, the minimum specific energy is $E_c = 3/2 y_c$, and the Froude number $F_r = 1$, where F_r is defined by

$$F_r = \frac{V}{\sqrt{g y}} \quad (3.11)$$

For supercritical flows, the Froude number is greater than 1, while for subcritical flows, the Froude number is less than 1.

Critical depth occurs when flow passes over broad-crested weirs, where the slope changes from a mild slope ($y_n > y_c$) to a steep one ($y_n < y_c$), and just upstream of a free overfall of a mild slope channel. Note that the critical depth is function only of discharge while the normal depth is function of depth and slope (Manning's equation).

3.3 Energy Equation

The one-dimensional energy equation for open-channels between two points is

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_t \quad (3.12)$$

Expressing the velocity in terms of the depth and the constant flow Q in the energy equation, the resulting equation is in terms of the depth expressed in a cubic or power form. The solution of that cubic equation gives three roots, one of which is nonphysical. The two other roots are the possible depth values for a given value of the energy: one large depth (subcritical) and one small depth value (supercritical). The depths are called alternate depths. The applicable depth for a particular problem is selected based on physical reasoning.

Flow in Open-Channels

3.4 Weirs

3.4.1 Broad-crested weirs

If the weir is long in the flow direction of flow so that the flow leaves the weir in essentially horizontal direction, the weir is a broad-crested weir. The flow depth is critical over the weir and the flow is thence given by

$$Q = L\sqrt{gy_c^3} \quad (3.13)$$

where L is the width of the weir. At critical flow conditions, the critical depth and the specific energy are

$$y_c = \frac{V_c^2}{g} \quad E = \frac{3}{2}y_c \quad (3.14)$$

Hence, the flow rate is

$$Q = \left(\frac{2}{3}\right)^{3/2} L\sqrt{gE^{3/2}} \quad (3.15)$$

where $E = H + V^2/2g$ is the total head above the crest. For negligible velocity of approach, the discharge is expressed as

$$Q = 0.385CL\sqrt{2g}H^{1.5} \quad (3.16)$$

where C is the resistance coefficient or discharge coefficient to account for the head loss and the shape of the weir ($C = 0.85-1.05$). The discharge coefficient can be considered as the ratio of the actual discharge over the theoretical discharge.

3.4.2 Sharp-crested weirs

Sharp-crested weirs are simple devices for discharge measurement in canals and flumes. Because the velocity is different at each point in the plane of the weir, a relationship between the velocity and the upstream head is needed. The relationship is derived from the energy equation when expressed between a point upstream and a point at a depth h in the plane of the sharp-crested weir.

The discharge over rectangular sharp-crested weir can be expressed in the following general form

$$Q = KL\sqrt{2g}H^{1.5} \quad K = \frac{2}{3}C_d \quad (3.17)$$

where K is the discharge coefficient, L is the effective length of the weir crest, and H is the measured head above the crest, *excluding* the velocity head. A discharge coefficient C_d must be applied to account for viscous effects and head loss.

For low flow rates, triangular weirs are used as they are more accurate than rectangular weirs. The discharge equation is also derived by integration

$$Q = \frac{8}{15}C_d\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{2.5} \quad (3.18)$$

For θ values of 60° and 90° , the flow coefficient K varied from 0.6 to 0.57 as the head varied from 0.2 to 2.0 ft.

3.5 Hydraulic Jump

The sequent depths across the hydraulic jump are derived from the momentum equation since the head loss term in the energy equation is difficult to quantify. The momentum equation gives

$$\sum F_x = \sum V_x \rho V \cdot A \quad (3.19)$$

Substituting the hydrostatic forces, eq. (3.19) becomes

$$\bar{p}_1 A - \bar{p}_2 A_2 = \rho V_1 (-A_1 V_1) + \rho V_2 (A_2 V_2) \quad (3.20)$$

For rectangular channels, the pressure at the centroids is $\bar{p} = \gamma \bar{y}$ where $\bar{y} = y/2$ and the sequent depths are therefore related through

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8F_{r1}^2} - 1 \right) \quad (3.21)$$

For trapezoidal channels, the centroids of the trapezoidal cross-section is

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{by \frac{y}{2} + my^2 \frac{y}{3}}{by + my^2} \quad (3.22)$$

The sequent depth is then calculated by iteration. The above derivation is for horizontal channels. A similar derivation for sloping channels can be developed by taking into account the longitudinal effect of weight of water mass.

The length of the hydraulic jump is estimated to be around $6y_2$ for $4 < F_{r1} < 20$. The head loss in a hydraulic jump in a rectangular channel is obtained from comparing the energy before and after the jump

$$h_l = \frac{(y_2 - y_1)^3}{4y_1 y_2} \quad (3.23)$$

3.6 Gradually Varied Flow

Water surface profiles are required in floodplain and channel design projects. They are instrumental in defining the levels of flood inundation. Profiles are sensitive to peak flows, cross-sectional geometry, bridge locations, and friction losses through the roughness coefficient. The water surface profiles classifications are: mild slope, steep slope, horizontal, critical, and adverse slope.

There are two methods: the direct step method and the standard step method. In the direct step method, the length of the reach Δx is calculated for a given water depth y . In the standard step method, the water depth y is computed at given location. The direct step method is applicable to prismatic channels (i.e. uniform cross-sections) while the standard step method is for all types of channels, natural or man-made, and it uses additional terms for the head losses in the channel expansion and contraction segments and kinetic energy correction terms.

Flow in Open-Channels

3.6.1 Direct Step Method

The length of the reach Δx is solved using the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \quad E = y + \frac{V^2}{2g} \quad (3.24)$$

where the friction slope is either given by

$$S_f = \frac{n^2 \bar{V}^2}{R^{4/3}} \quad \text{or} \quad S_f = \frac{\bar{Q}^2}{\bar{K}^2} \quad (3.25)$$

where $K = AR^{2/3}/n$.

3.6.2 Standard Step Method

The water surface profile is computed from the energy equation in the following form

$$y_1 + \frac{\alpha_1 V_1^2}{2g} + z_1 = y_2 + \frac{\alpha_2 V_2^2}{2g} + z_2 + S_f \Delta x + C_l \left| \frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right|$$

Because of the large difference in velocities between the floodway and the banks, a kinetic-energy correction factor is used to account for the large velocity variation within a cross-section

$$\alpha = \frac{A_T^2}{K_T^3} \sum_{i=1}^N \left(\frac{K_i^3}{A_i^2} \right) \quad (3.26)$$

where K_i is the conveyance factor ($K = AR^{2/3}/n$) in subsection i and N is the total number of subareas. The coefficient C_l is the expansion/contraction coefficient and the friction slope is $S_f = Q^2/\bar{K}^2$, where the mean conveyance factor \bar{K} can be calculated in two different ways: arithmetic mean and harmonic mean. The arithmetic mean is

$$\bar{K} = \frac{K_1 + K_2}{2} \quad (3.27)$$

The harmonic mean is obtained from the definition of the mean friction slope $\bar{S}_f = (S_{f1} + S_{f2})/2$. Hence

$$\frac{1}{\bar{K}^2} = \frac{1}{2} \frac{K_1^2 + K_2^2}{K_1^2 K_2^2} \quad (3.28)$$

3.7 Flood Plain Delineation

Flood plain delineation involves the definition of the extent of the flood plain for a particular design flood, i.e. the area of the flood plain that will be inundated by the design flood. The floodplain is divided into a major floodway with a large velocity component and the peripheral area that acts as overbank flood storage. The delineation of the 100-yr floodplain involves the computation of the water surface profile along the floodway using the methods of gradually varied flow. Once the depth is determined, the extent of the flooded areas can be established on a contour map of the area.

3.8 Design of Channel Transitions

A common hydraulic structure is one that conveys water from one channel to another of a different shape. Such structures are called transition structures. For example, a transition structure is used between a trapezoidal channel and a rectangular flume to convey water around a steep hill.

There are three types of transition structures that are of the gradual type: cylinder-quadrant, wedge, and warped-wall. The former two are best suited for outlet transition (expansion) while all three are good for inlet transition (contraction). The recommended contraction and expansion angle for the wedge transition are 27.5° and 22.5° , respectively, while for the warped-wall, it is 12.5° for both the inlet and expansion.

The head loss for an inlet transition is $K_I V^2 / 2g$ where V is the highest velocity, while for an outlet expansion the head loss is $K_E (V_1^2 - V_2^2) / 2g$ where V_1 is the higher velocity.

For a given discharge and flow depth in the channel, the design of transition structures involves the following steps:

- Dimension the flume using the Froude criterion
- Size the length of the transition structure based on the angle of contraction or expansion
- Determine the invert elevation of the flume using the energy equation with the head loss
- Compute the water surface profile within the transition structure assuming proportional head loss and a linear variation in width of the contraction or the expansion.

Flow in Open-Channels

Example 3.1: Flow regime

Water is flowing uniformly in an open channel of triangular shape at a rate of $14 \text{ m}^3/\text{s}$. The channel has side slopes of 1:2 (V:H) and a roughness coefficient of $n = 0.015$. Determine whether the flow is subcritical or supercritical if the bottom slope is 5 m/km.

Solution

The uniform depth is given by Manning's equation

$$14 = \frac{1}{0.015} m y^2 \left(\frac{m y^2}{2y\sqrt{1+m^2}} \right)^{2/3} \sqrt{0.005}$$

where $m = 2$. Solving for y , one obtains $y = 1.42 \text{ m}$

The critical depth is given by

$$\frac{TQ^2}{gA^3} = \frac{(14)^2 (2my)}{9.81 (my^2)^3} = 1$$

Solving for y , one gets $y = 1.585 \text{ m}$. The uniform depth is smaller than the critical depth; therefore, the flow is supercritical.

Example 3.2: Flow over an upstep

Given a discharge of $30 \text{ m}^3/\text{s}$ in a horizontal rectangular channel of width of 3 m, compute the depth of flow at the 30 cm high upstep location A given a uniform depth of 3 m at the upstream point B. What are the depths at points A and B for an upstep of 50 cm height. Neglect the effect of friction.

Solution

Expressing the energy equation between B and A

$$z_B + y_B + \frac{V_B^2}{2g} = z_A + y_A + \frac{V_A^2}{2g}$$

One can solve for the depth y_A for $q = 30/3$, $y_B = 3$, $V_B = 30/9$, $z_B = 0$, and $z_A = 0.3$ noting that $V_A = 10/y_A$. The depth at location A is 2.315 m.

For an upstep of 50 cm, similar calculation yields a value of $y_A = 2.086 \text{ m}$, which is less than the critical depth $y_c = \sqrt[3]{10^2/9.81} = 2.168 \text{ m}$. This is not possible physically since the depth on the hump A must remain at the critical depth y_c for any increase in the upstep height Δz above the critical hump height. Hence, a damming effect occurs and the flow depth at point B increases from 3 to a depth as given by the following energy equation

$$y_B + \frac{10^2}{2gy_B^2} = 0.5 + 2.168 + \frac{10^2}{2g(2.168)^2}$$

The flow depth at point B is therefore $y_B = 3.278 \text{ m}$.

Example 3.3: Water surface profile – H2

A horizontal rectangular concrete channel terminates in a free outfall. The channel is 4 m wide and carries a discharge of water of 12 m³/s. Compute the water depth 300 m upstream from the outfall. The Manning’s roughness coefficient n is 0.013.

Solution

The critical depth at the outfall is

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{9}{9.81} \right)^{1/3} = 0.972$$

The location of the critical depth is at $3y_c$ from the outfall. The length of the reach Δx is obtained from the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \qquad E = y + \alpha \frac{V^2}{2g} \qquad S_f = \frac{Q^2}{K^2} \qquad K = \frac{1}{n} AR^{2/3}$$

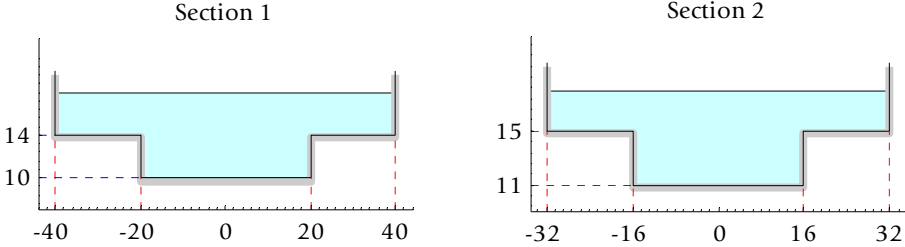
Here $\alpha \approx 1$, $n = 0.013$, and $S_0 = 0$ for a horizontal concrete channel. The water surface profile is calculated using the direct step method since the channel configuration is uniform. The downstream depth is given by the critical depth of 0.97 m.

Section	Depth	A	P	R	K	V	E	K_m	S_f (10^{-3})	Δx	x
1	0.9717	3.887	5.943	0.654	225.3	3.087	1.458	0	0	0	0
2	1	4	6	0.6667	234.8	3	1.459	230	2.721	-0.4378	-0.4378
3	1.1	4.4	6.2	0.7097	269.3	2.727	1.479	252.1	2.267	-8.995	-9.433
4	1.2	4.8	6.4	0.75	304.8	2.5	1.519	287	1.748	-22.57	-32
5	1.3	5.2	6.6	0.7879	341.2	2.308	1.571	323	1.38	-38.31	-70.32
6	1.4	5.6	6.8	0.8235	378.5	2.143	1.634	359.8	1.112	-56.3	-126.6
7	1.5	6	7	0.8571	416.5	2	1.704	397.5	0.9115	-76.61	-203.2
8	1.6	6.4	7.2	0.8889	455.1	1.875	1.779	435.8	0.7582	-99.33	-302.6

Flow in Open-Channels

Example 3.4: Water Surface Profile Computation – Standard Step Method

Compute the water surface profile at the upstream section 2 given that the water surface elevation at section 1 is 18 m. The river is subject to a flood of 300 m³/s. The distance between the two sections is 4 km and Manning’s *n* is 0.045 in the overbank areas and 0.025 in the main channel. Use expansion and contraction coefficients of 0.3 and 0.1, respectively.



Solution

The water surface profile is computed from the energy equation in the following form

$$y_1 + \frac{\alpha_1 V_1^2}{2g} + z_1 = y_2 + \frac{\alpha_2 V_2^2}{2g} + z_2 + S_f \Delta x + C_l \left| \frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right|$$

where $S_f = Q^2 / \bar{K}^2$, $K = AR^{2/3} / n$, $C_l = C_e$ is the expansion coefficient and

$$\alpha = \frac{A_T^2}{K_T^3} \sum_{i=1}^N \left(\frac{K_i^3}{A_i^2} \right)$$

The detailed solution is tabulated below noting that ΔH denotes the difference in the total head between section 1 and 2. The head losses are shown as negative quantities because they are tabulated alongside the results for section 2 (upstream). This is an artifact of the computer program.

Design of Channel Transitions

Sec	Iter	x	w	z	y	B	m	n	A	P	R	K
												10^3
1		0	18	14	4	20	0	0.045	80	24	3.333	3.967
				10	8	40	0	0.025	320	48	6.667	45.34
				14	4	20	0	0.045	80	24	3.333	3.967
2	1	-4000	20	15	5	16	0	0.045	80	21	3.81	4.336
				11	9	32	0	0.025	288	40	7.2	42.95
				15	5	16	0	0.045	80	21	3.81	4.336
2	2	-4000	19	15	4	16	0	0.045	64	20	3.2	3.088
				11	8	32	0	0.025	256	40	6.4	35.3
				15	4	16	0	0.045	64	20	3.2	3.088
2	3	-4000	18.5	15	3.5	16	0	0.045	56	19.5	2.872	2.514
				11	7.5	32	0	0.025	240	40	6	31.7
				15	3.5	16	0	0.045	56	19.5	2.872	2.514
2	4	-4000	18.2	15	3.2	16	0	0.045	51.2	19.2	2.667	2.188
				11	7.2	32	0	0.025	230.4	40	5.76	29.61
				15	3.2	16	0	0.045	51.2	19.2	2.667	2.188
2	5	-4000	18.17	15	3.17	16	0	0.045	50.72	19.17	2.646	2.156
				11	7.17	32	0	0.025	229.4	40	5.736	29.41
				15	3.17	16	0	0.045	50.72	19.17	2.646	2.156

w	A_T	K_T	K_m	Q_m	S_f	h_l	α	V	KE	h_e	H	ΔH
		10^3	10^3		10^{-3}	10^{-3}				10^{-3}		
18	480	53.27	0	300	0	0	1.417	0.6250	0.0282	0	18.03	0
20	448	51.63	52.45	300	0.0327	-130.9	1.431	0.6696	0.0327	-1.349	20.03	-1.872
19	384	41.48	47.37	300	0.0401	-160.4	1.417	0.7813	0.0441	-4.76	19.04	-0.851
18.5	352	36.73	45	300	0.0444	-177.8	1.408	0.8523	0.0521	-7.18	18.55	-0.339
18.2	332.8	33.99	43.63	300	0.0473	-189.1	1.402	0.9014	0.0581	-8.963	18.26	-0.032
18.17	330.9	33.72	43.5	300	0.0476	-190.3	1.402	0.9067	0.0587	-9.158	18.23	-0.001

Here H is the hydraulic head and ΔH is the difference in head, i.e. $\Delta H = H_u - H_d - h_l - h_e$.

Flow in Open-Channels

Example 3.5: Design of a warped-wall outlet transition

Design a warped-wall outlet transition ($K=0.3$ & $\theta=12.5^\circ$) to join a concrete flume of rectangular cross section and a trapezoidal concrete-lined channel. The Froude number in the flume is to be equal to or less than 0.6. The Manning's roughness coefficient is 0.015 and the flow in the channel is 125 cfs. The trapezoidal channel is on a slope of 0.0002 and has side slopes of 1 vertical to 2 horizontal. The bottom width of the trapezoidal channel is 8 ft and its invert elevation is 100 ft. Set the flow depth in the flume equal to the width of the flume.

Solution

- Using Manning's equation, the normal depth is 3.49 ft for a discharge of 125 cfs.
- The Froude number is given by

$$F_r = \frac{V}{\sqrt{gy}} = \frac{Q}{by\sqrt{gy}} = 0.6$$

- Setting $b = y$, one obtains $y = 4.22$ ft.
- Selecting $b = 4.25$ ft, one gets $y = 4.2$ ft (One can also choose $b = 5$ ft and get another y)
- The length L based on the angle of 12.5° for warped-wall outlets is

$$L = \frac{4 + 2(3.49) - 0.5(4.25)}{\tan(12.5^\circ)} = 40 \text{ ft}$$

- The invert elevation is obtained from the energy equation using $k = 0.3$ for the head loss

$$y_1 + z_1 + \frac{V_1^2}{2g} = y_2 + z_2 + \frac{V_2^2}{2g} + \frac{k}{2g}(V_1^2 - V_2^2)$$

- Solving for z_1 using $y_1 = 4.205$, $y_2 = 3.49$, $V_1 = 125/(4.25)^2 = 6.98$ and $V_2 = 125/A_2 = 2.39$ with $A_2 = 3.49(8 + 2 * 3.49) = 52.28$, one gets $z_1 = 98.8$ ft.

Appendix

Appendix

Key Exam Problems

- Part I: Pipe Flow Problems
 - Determine the head loss, given the flow rate, kind and size of pipe.
 - Determine the flow rate, given the head, kind and size of pipe
 - Determine the size of pipe, given the head, flow rate, and kind of pipe.
 - Compute the pump power needed to drive a system
 - Compute the power loss or gain
 - Calculate the minor loss coefficient of a valve or other
 - Compute the discharge in a parallel (or series) pipe system
 - Find the spots of low pressures or minimum pressure
 - Locate all the minor losses in the system
 - Compute the flow rates and elevation heads in a three reservoir system
 - Compute the time to fill a tank
 - Compute the flow rates in a pipe manifold system
 - Compute the flow rates in a network
- Part II: Surface Flow Problems
 - Compute the uniform flow in a trapezoidal or circular channel given the depth of flow
 - Compute the depth of flow in a trapezoidal or circular channel assuming uniform flow
 - Compute the allowable encroachment for a specific increase in depth
 - Compute the critical depth for any channel geometry
 - Determine the flow regime at a section: subcritical or supercritical
 - Compute the flow rates over broad-crested and sharp-crested weirs
 - Compute the depth upstream given the depth downstream or vice versa
 - Compute the depth downstream or upstream of a hydraulic jump
 - Compute the energy loss in a hydraulic jump
 - Determine the types of water surface profiles in a channel
 - Compute the water surface profile using the direct step method
 - Compute the water surface profile using the standard step method
- Part III: Design Problems
 - Design the best hydraulic section
 - Design a transition structure
 - Design the depth of a stilling basin
- EPANET & HECRAS software

Technical Reports Guidelines

Students are expected to present their laboratory and project reports in a professional manner with correct spelling, grammar and syntax. This is not to impose unnecessary work on the student but rather to become accustomed to the requirements of the professional world.

The report should be typed using a word processor and printed on A4 paper with at least 2.5 cm margin at the top, bottom and sides. The text must be in 11 or 12-point type and the headings in 14-point type with 1.5 line spacing. The font should be Times New Roman, Arial or similar. The headings are numbered according to the decimal system. Each main heading is a whole number and the sub-heading is numbered as a decimal of that. The use of the MS equation editor is encouraged. Add one extra line space above and below all displayed equations. When an experiment consists of several parts, the results of each part should be given on a separate page. Each page of the report should be numbered in the top or bottom right-hand corner. The title page must include the title of the report, the authors' name, group number, date and place.

The computations should be done in orderly steps with all assumptions clearly stated and their source given. All calculations should be reproducible. Use of computer programs such as EXCEL or MATLAB is encouraged. All graphs should be produced using graphics software. The figures must have a caption and axis labels and they must fit the size of half a page. Each table must have a title, and all columns must have headings. All figures and tables must be cited in the text.

The reference list should be explicit with the author's name, title, publisher and date. References should be correctly cited in the text by giving the authors' name and date of publication [e.g. *Roberson & Crowe*, 1995]. Acknowledgments should be duly conferred and copied material should be duly credited.

The lab report structure should include:

- Title page
- Table of Contents
- List of Figures
- List of Tables
- Abstract. (1–2 ¶)
- Introduction. (1–2 ¶)
- Theory & Assumptions: Underlying principle, governing equations & assumptions (2–3 ¶)
- Experimental Procedure: Stepwise method explaining what you did and why you did it.
- Data & Measurements: Tabular form
- Calculations & Results: 1 sample calculation. Show all results in tabular & graphical form.
- Discussion and Analysis: Comments on the results. (2–4 ¶)
- Conclusions. (1–2 ¶)
- Acknowledgments.
- References: Books, lecture notes, etc.
- Appendix: Any relevant material but of secondary importance.

The report is graded as follows:

- | | |
|--|-----|
| ▪ Structure & Format: esp. Cover Page, Figure Captions, Table Titles, References | 4 |
| ▪ Background Material: Introduction and Theory | 4 |
| ▪ Numerical Results: Accuracy and Completeness | 4 |
| ▪ Analysis, Discussion, and Conclusion | 4 |
| ▪ Presentation & Aesthetic: Figures, Tables and Overall Appearance | 4 |
| ▪ Originality of style | ± 2 |

N.B.: The first lab report is a group report while the second report is an individual report. The project report is to be submitted by the group.

Appendix

Lab Sign-up Form

Lab Sessions	Slot	Time	Monday	Tuesday	Wednesday	Thursday	Friday
Session 1	M 11:00 – 11:45	08:00					Session 09
Session 2	M 11:45 – 12:30	08:45					Session 10
Session 3	M 12:30 – 13:15	09:30					Session 11
Session 4	M 13:15 – 14:00	10:15					Session 12
Session 5	M 14:00 – 14:45	11:00	Session 01				Session 13
Session 6	M 14:45 – 15:30	11:45	Session 02				Session 14
Session 7	M 15:30 – 16:15	12:30	Session 03				Session 15
Session 8	W 16:15 – 17:00	13:15	Session 04				Session 16
Session 9	F 08:00 – 08:45	14:00	Session 05				Session 17
Session 10	F 08:45 – 09:30	14:45	Session 06				Session 18
Session 11	F 09:30 – 10:15	15:30	Session 07				Session 19
Session 12	F 10:15 – 11:00	16:15	Session 08				Session 20
Session 13	F 11:00 – 11:45	17:00					
Session 14	F 11:45 – 12:30						
Session 15	F 12:30 – 13:15						
Session 16	F 13:15 – 14:00						
Session 17	F 14:00 – 14:45						
Session 18	F 14:45 – 15:30						
Session 19	F 15:30 – 16:15						
Session 20	F 16:15 – 17:00						

Group #:

Group Name:

Students' Names:

Date

Session

Experiment 1: Fluid Friction & Minor Losses

Experiment 2: Sharp-Crested Weirs

Experiment 3: Ogee Spillway & Hydraulic Jump

Project Sign-Up Form

The objectives of the Hydraulics project are twofold:

1. Use of the technical software EPANET or HEC-RAS to carry out a hydraulic analysis
2. Presentation of the results in a professional report

The project can be a typical homework problem or a practical hydraulics project. However, students should note well that the discussion of a homework problem is limited, while there is a lot in a practical problem that can be discussed, e.g. engineering solutions, effect on society, economical impact.

A written report is to be submitted along with the input and output file of the software.

Group #:

Group Name:

Students' Names:

Project Title:

Dates

Deliverable 1:

Deliverable 2:

Deliverable 3:

Appendix

Judgment Day Sign-Up Schedule

Sign-up your group name. Any absent group member results in a group penalty.

Have your project on the laptop ready.

Thursday 10-Jan-13	Thursday 10-Jan-13	
11:00	14:00	
11:20	14:20	
11:40	14:40	
12:00	15:00	
12:20	15:20	
12:40	15:40	
13:00	16:00	
13:20	16:20	
13:40	16:40	

Grading Guidelines – Laboratory Reports

I. Structure & Format of the Report (-0.5 for each item missing)

1. Title page includes: title of report, name of author, group number, date and place
2. Headings, table titles, and figure captions are appropriate
3. Sections and sub-sections have consistent numbering
4. Each table and figure is cited in the text
5. All graphs have a caption, axis labels and fit half a page (or less)
6. All tables have a title, and all columns have headings
7. The reference list is explicit with the author's name, title, publisher and date
8. All pages are numbered

II. Background Material (-0.5 for each faulty item; -1 for each conceptual mistake)

1. All equations are correct
2. All terms are defined
3. All assumptions are valid
4. The student uses his own words to describe the experimental procedure

III. Data & Calculations (-0.5 for each calculation mistake; -1 for each conceptual mistake)

1. The values provided in the report match the raw data sheet values
2. All calculations are correct
3. All calculations are supported by sample calculations

IV. Analysis & Discussion (-0.5 for each faulty item; -1 for each conceptual mistake)

1. The student has a good understanding of the theory
2. The student tries to explain the error based on his knowledge of the course material
3. The student makes an effort in exploring the outside literature
4. The student has common sense

V. Aesthetics (-0.5 for each flawed item)

1. All figures and tables are necessary
2. All figures have good resolution
3. All equations are typed using MS Equation
4. A line is left blank above and below each table, figure and equation
5. No more than 3 significant figures are used in tables and calculations
6. The overall appearance of the report is acceptable and professional

VI. Originality of Style (± 2)

1. Shamelessly copied (-2)
2. Copied with finesse (-1)
3. Neutral (0)
4. Nice (1)
5. Outstanding (2)

Appendix

Grading Guidelines – Project Reports

I. Structure & Format of the Report (-0.5 for each item missing)

1. Title page includes: title of report, name of author, date and place
2. Headings, the list of tables, and the list of figures are relevant
3. All graphs have a caption, axis labels and fit half a page (or less)
4. All tables have a title, and all columns have headings
5. Each figure and table is cited in the text
6. The reference list is explicit with the author's name, title, publisher and date
7. All pages are numbered and the Table of Content is updated
8. The report is concise and not excessive (less than 20 pages)

II. Background Material (-0.5 for each faulty item; -1 for each conceptual mistake)

1. The engineering problem is defined (Introduction)
2. The theoretical background is covered (Theory)
3. The methodology is described (Methods)
4. The group spent an individual effort on the above

III. Data & Calculations (-0.5 for each error; -1 for each conceptual mistake)

1. The data collected or inputted is logical (Input)
2. The results are original, complete, and consistent (Output)

IV. Analysis & Discussion (-0.5 for each faulty item; -1 for each conceptual mistake)

1. There is a good understanding of the theory
2. The analysis is correct and complete (with well-described limitations)
3. The key findings of the project are well-communicated (Summary and Conclusions)
4. There is an effort in exploring the outside literature

V. Professional Look (-0.5 for each flawed item)

1. All figures have a good resolution
2. All figures and tables are necessary
3. The report is supported with appropriate pictures and other visuals
4. The font and style of the report is uniform throughout
5. The page layout is consistent throughout the report
6. The English content is correct (spelling, grammar, and syntax)
7. The English style is flowing, clear, and concise
8. The overall appearance of the report is professional

VI. Originality of Style (± 2)

1. Shamelessly copied (-2)
2. Copied with finesse (-1)
3. Neutral (0)
4. Nice (1)
5. Eye-catching (2)

Other Comments:

Title – Authors:

Exam Rules

- Seating will be assigned and posted 20 minutes before the exam.
- You must show up with only a pen or a pencil.
- Any excess baggage is subject to the penalties shown below.
-
- The duration of the exam is 89 minutes.
- Papers will not be collected by the proctors at the end of the exam period.
- It is the responsibility of the examinee to submit the paper on time.
- Any delay in submitting the paper would be subject to a penalty of 1 point per minute.
-
- No questions allowed. Permission to ask a question costs half a point.
- A question to a proctor costs 1 point.
- An answer by the GA proctor costs 2 points to the examinee and the GA.
-
- Whispers are subject to immediate expulsion from the exam room.
- Students who resist expulsion would be subject to the full university disciplinary action.
-
- All problems are of equal weight with 10 points each.
- Answers should be systematic and clear. No partial credit for scrap writing.
- There is a penalty for conflicting answers or irrelevant write-up.
-
- Make all necessary assumptions when needed.
- Use at least 4 significant digits in your calculations.
- Show all calculations. No credits for absent intermediate calculations.
-
- Write your answers on the question sheet.
- You can use both sides of the page and an extra sheet is provided for each problem.
- A list of necessary and unnecessary formulae is also provided (see sample exam).
-
- Write your name and sign the pledge of honor (see sample exam).
- A missing name results in a penalty of 2 points.
- The grade of a paper with a missing signature will not be released.
-
- Do not hurry, panic, or fret. Just do better than your neighbor.
- Good luck!

Excess Exam Baggage – Penalty List

Cellular – On	–6 points
Cellular – Off	–3 points
Programmable Calculator – On	–8 points
Programmable Calculator – Off	–4 points
Books/Notes – Open	–10 points
Books/Notes – Closed	–5 points
Handbags – Open	–4 points
Handbags – Closed	–2 points
Pencil Cases – Open & Closed	–1 point
Cigarettes	–1 point
Drinks	–2 points
Jackets	–2 points
Unidentified Object: Flying & Non-Flying	–5 points

Nota Bene:

- The above penalties apply to all areas surrounding the student, i.e. beneath the seat, side table, table behind, and the airspace above.
- Penalties are additive: an open handbag with a cellular on and a closed book is –15 points
- The maximum deduction is 30 points.
- Service and handling charges are included.

Homework Problems 2006

HomeWork I

NAME:

ID #:

The first 95 miles of the Delta-Mendota Canal in California is a trapezoidal channel with bottom width of 48 ft and side slopes of 1:1.5 (V:H). The channel is designed to carry a flow of 4600 cfs at a depth of 16.7 ft. How much would the best hydraulic section increase the flow capacity for the same cross-sectional area and side slopes? What would be the corresponding depth and bed width?

Solution

The optimum cross-section for the same cross-sectional area A is obtained from $\partial Q/\partial y = 0$ or $\partial R/\partial y = 0$ where $R = A/P$, $A = y(b + my)$ and $P = b + 2y\sqrt{1 + m^2}$.

Expressing R in terms of y and A by eliminating b , one obtains

$$R = \frac{A}{\frac{A}{y} + y(2\sqrt{1+m^2} - m)} = \frac{A}{\frac{A}{y} + Cy}$$

Differentiating

$$\frac{\partial R}{\partial y} = -\frac{A(C - A/y^2)}{\left(\frac{A}{y} + Cy\right)^2} = 0$$

Hence

$$y = \sqrt{\frac{A}{C}} = \frac{\sqrt{A}}{\sqrt{2\sqrt{1+m^2} - m}}$$

The existing cross-sectional area is $A = 16.7(48 + 1.5 \times 16.7) = 1220 \text{ ft}^2$, the wetted perimeter is $P = 48 + 2(16.7)\sqrt{3.25} = 108.2 \text{ ft}$ and the hydraulic radius is $R = 11.27 \text{ ft}$.

The optimum depth is computed as $y = 24.07 \text{ ft}$. Hence, the bed width is 14.56 m and the optimum R is 12.04 ft. The optimum flow rate is therefore

$$Q_o = Q \left(\frac{R_o}{R}\right)^{2/3} = 4600 \left(\frac{12.04}{11.27}\right)^{2/3} = 4807$$

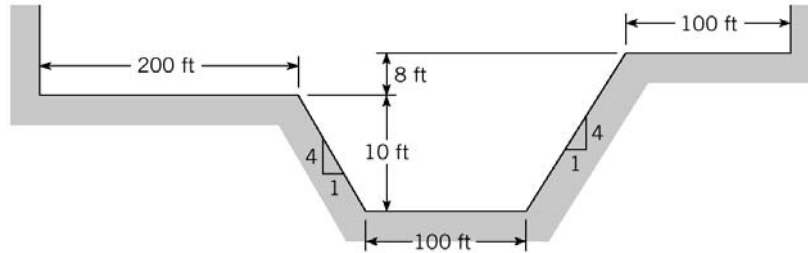
This is 4.5% increase on the existing flow conditions.

HomeWork II

NAME:

ID #:

The compound cross section shown below has Manning's n values of 0.025 for the main channel and 0.070 for the overbank areas. Determine the minimum width allowable for a 100-yr flood flow of 80,000 cfs that will not increase the depth by more than 1 ft. The channel has a uniform slope of 0.004.



Solution

The depth for a flow of 80,000 cfs is obtained by applying Manning's equation for a compounded section.

The uniform flow at full depth in the main channel is 16176 cfs. The uniform flow in the main channel and the left overbank area for a depth of 18 ft is 50413 cfs. The flow at depth of 20 ft is 63000 cfs and the flow for a depth of 22.4 ft is around 80,000 cfs.

Depth (ft)	10	18	20	22.4
Flow (cfs)	16176	50413	63000	80000

Increasing the flow depth by 1 ft to 23.4 ft, the amount of width reduction for the same flow rate is around 120 ft for equal encroachment width on both overbanks. The combined flow rate for a total width of $100 + 300 + 28/4 - 120 = 287$ ft is 79,891 cfs.

For an encroachment on the left overbank only, the width encroached is 73 ft and the total width available is $407 - 73 = 334$ ft. The discharge rate is then 80,072 cfs.

HomeWork III

NAME:

ID #:

A rectangular channel with $n = 0.012$ is 5 ft wide and is built on a slope of 0.0006 ft/ft. At point A, the flow rate is 60 cfs and the depth is 3 ft. Use the direct step method to find the distance to point B where the depth is 2.5 ft and determine whether this point is upstream or downstream of point A.

Solution

The normal depth is 3.16 ft and the critical depth is 1.65 ft; the flow is therefore subcritical ($y > y_c$) and the slope is mild ($y_n > y_c$). Since the depth range is between y_n and y_c , the water surface follows the M2 profile. Hence, the integration proceeds upstream from the lower depth of 2.5 (point B) to the higher depth of 3.0 (point A).

The results for $\Delta y = 0.05$ is

Section	Depth (ft)	Area (ft ²)	R (ft)	Velocity (ft/s)	Mean V (ft/s)	Mean R (ft)	S (10 ³ ft/ft)	Δx (ft)	x (ft)
1	2.50	12.50	1.25	4.80					0
2	2.55	12.75	1.26	4.71	4.75	1.26	1.081	-75	-75
3	2.60	13.00	1.27	4.62	4.66	1.27	1.026	-87	-162
4	2.65	13.25	1.29	4.53	4.57	1.28	0.975	-100	-262
5	2.70	13.50	1.30	4.44	4.49	1.29	0.928	-117	-379
6	2.75	13.75	1.31	4.36	4.40	1.30	0.883	-137	-516
7	2.80	14.00	1.32	4.29	4.32	1.32	0.842	-163	-680
8	2.85	14.25	1.33	4.21	4.25	1.33	0.803	-197	-877
9	2.90	14.50	1.34	4.14	4.17	1.34	0.767	-243	-1120
10	2.95	14.75	1.35	4.07	4.10	1.35	0.733	-308	-1428
11	3.00	15.00	1.36	4.00	4.03	1.36	0.702	-409	-1836

The results for $\Delta y = 0.1$ is

Section	Depth (ft)	Area (ft ²)	R (ft)	Velocity (ft/s)	Mean V (ft/s)	Mean R (ft)	S (10 ³ ft/ft)	Δx (ft)	x (ft)
1	2.50	12.5	1.25	4.80					0
2	2.60	13.0	1.27	4.62	4.71	1.26	1.054	-161	-161
3	2.70	13.5	1.30	4.44	4.53	1.29	0.951	-216	-377
4	2.80	14.0	1.32	4.29	4.37	1.31	0.863	-299	-678
5	2.90	14.5	1.34	4.14	4.21	1.33	0.785	-435	-1111
6	3.00	15.0	1.36	4.00	4.07	1.36	0.718	-702	-1813

HomeWork IV

NAME:

ID #:

A hydraulic jump occurs in a trapezoidal channel. The velocity and the depth upstream of the jump are 10 m/s and 40 cm, respectively. Determine the depth of flow downstream of the jump. The bottom width of the channel is 5 m and the side slopes are 1:1.

Solution

The momentum equation gives

$$\bar{p}_1 A + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2$$

The pressure at the centroids is $\bar{p} = \gamma \bar{y}$ where

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{by \frac{y}{2} + my^2 \frac{y}{3}}{by + my^2}$$

Hence, the momentum equation becomes

$$\gamma \left(by_1 \frac{y_1}{2} + my_1^2 \frac{y_1}{3} \right) + \rho Q V_1 = \gamma \left(by_2 \frac{y_2}{2} + my_2^2 \frac{y_2}{3} \right) + \rho Q V_2$$

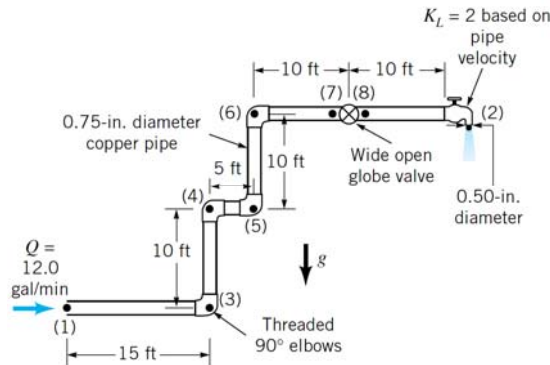
The upstream cross-sectional area is $A = by + my^2 = 5(0.4) + (0.4)^2 = 2.16 \text{ m}^2$. The flow rate is therefore $Q = VA = 21.6 \text{ m}^3/\text{s}$. Substituting the known values and solving for y_2 iteratively, one obtains $y_2 = 2.45 \text{ m}$.

HomeWork V

NAME:

ID #:

Water flows from the basement to the second floor through the 0.75-in. diameter copper pipe ($k_s = 5 \times 10^{-6}$ ft) at a rate of $Q = 12$ gal/min $= 0.0267$ ft³/s and exits through a faucet of diameter 0.50 in. as shown in the Figure. Determine the pressure head drop in ft from point 1 to point 2. The fluid properties are $\gamma = 62.4$ lb/ft³ and $\nu = 1.21 \times 10^{-5}$ ft²/s.



Solution

Expressing the energy equation between point 1 and point 2, one obtains

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(\sum f \frac{L}{D} + \sum k_m + K_L \right) \frac{V_1^2}{2g}$$

The pressure drop is then

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + z_2 - z_1 + \left(\sum f \frac{L}{D} + \sum k_m + K_L \right) \frac{V_1^2}{2g}$$

Here $z_2 - z_1 = 20$ ft and $\sum k_m = 4(0.9) + 10$. The total length is $L = 60$ ft.

The velocity in the pipe is $V_1 = Q/A = 8.7$ ft/s and in the faucet it is $V_2 = 19.6$ ft/s. The pipe friction factor for $R_e = VD/\nu = 45000$ is from

$$f = \frac{0.25}{\left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^2} = 0.0216$$

Hence,

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = 4.79 + 20 + \left(.0216 \frac{60}{0.75/12} + 13.6 + 2 \right) 1.175 = 67.5 \text{ ft}$$

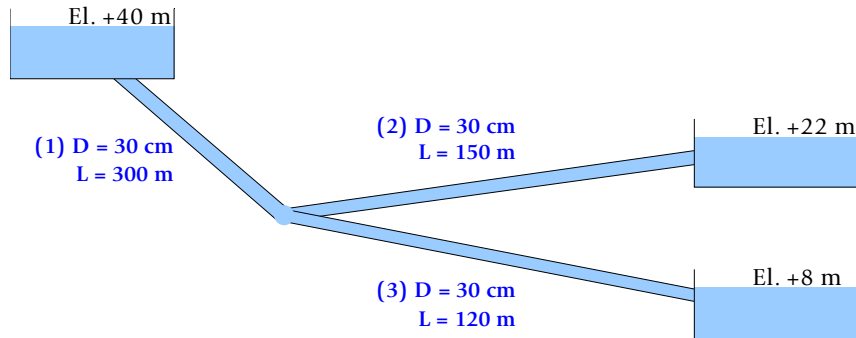
The pressure drop is 67.5 ft.

HomeWork VI

NAME:

ID #:

Determine the flow rate into or out of each reservoir. Use $f = 0.020$



Solution

Expressing the energy equation between reservoir 1 and the junction, one obtains

$$40 = H_j + \left(0.5 + 0.020 \frac{300}{0.3} \right) \frac{V_1^2}{2g}$$

Expressing the energy equation between reservoir 3 and the junction, one obtains

$$H_j = 8 + \left(1 + 0.020 \frac{120}{0.3} \right) \frac{V_3^2}{2g}$$

Assuming that $H_j = 22$, one obtains $V_1 = 4.15$ m/s and $V_3 = 5.53$ m/s. Hence, the head at the junction must be smaller than 22 and the third reservoir equation is

$$22 = H_j + \left(0.5 + 0.020 \frac{150}{0.3} \right) \frac{V_2^2}{2g}$$

Finally, the continuity equation at the junction is $V_1 + V_2 = V_3$. Substituting the velocity terms in the continuity equation, one gets

$$\sqrt{\frac{2g(40 - H_j)}{0.5 + 0.020 \frac{300}{0.3}}} + \sqrt{\frac{2g(22 - H_j)}{0.5 + 0.020 \frac{150}{0.3}}} - \sqrt{\frac{2g(H_j - 8)}{1 + 0.020 \frac{120}{0.3}}} = 0$$

Or

$$\sqrt{0.957(40 - H_j)} + \sqrt{1.869(22 - H_j)} - \sqrt{2.18(H_j - 8)} = 0$$

Solving the above equation by iteration, one obtains $H_j = 21.29$ m. The velocities are then $V_1 = 4.23$ m/s, $V_2 = 1.15$ m/s and $V_3 = 5.38$ m/s. The flow rates are therefore $Q_1 = 0.3$ m³/s, $Q_2 = 0.08$ m³/s and $Q_3 = 0.38$ m³/s using $A = 0.0707$.

HomeWork VII

NAME:

ID #:

Design the size of pipe needed to carry water from the water main at a rate of $0.025 \text{ m}^3/\text{s}$ to a factory that is 160 m away. The pressure at a water main is 300 kPa gage and the pressure required at the factory is 60 kPa at a point 10 m above the main connection. Use a galvanized-steel pipe with $k_s = 0.15 \text{ mm}$ and $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$.

Solution

The energy equation yields

$$\frac{300}{9.81} + \frac{V^2}{2g} + 0 = \frac{60}{9.81} + \frac{V^2}{2g} + 10 + f \frac{160}{D} \frac{V^2}{2g}$$

Using the friction factor formula and expressing the velocity in terms of the flow rate and the unknown diameter, one can iterate on D until the energy equation is satisfied. The results for various trial runs are tabulated below

D (cm)	A (m^2)	V (m/s)	Re	f	y_1 (m)	y_2 (m)	h_i (m)	h_e (m)	h_L (m)	ΔE_B (m)
	10^3		10^6							
5	1.963	12.73	0.6366	0.02647	30.58	16.12	699.9	0	699.9	-685.5
10	7.854	3.183	0.3183	0.02257	30.58	16.12	18.65	0	18.65	-4.181
12	11.31	2.210	0.2653	0.02183	30.58	16.12	7.25	0	7.25	+7.215
10.5	8.659	2.887	0.3032	0.02236	30.58	16.12	14.47	0	14.47	-0.0098

HomeWork VIII

NAME:

ID #:

Determine the water surface elevation required in the upstream reservoir and the total discharge in the pipe manifold system if the discharge rate at the last port is 0.06 m³/s. The diameter of the manifold pipe is 40 cm and the diameter of the ports is 10 cm. There are 3 discharge openings that are 15 m spaced apart. The manifold is discharging into a water body at a depth of 5 m and the distance from the reservoir to the first opening is 500 m. Use $C_d = 0.675$ for the ports, $k_s = 0.35$ mm for the pipe & $\nu = 1.31 \times 10^{-6}$ m²/s.

Solution

The flow rates in the manifold system is obtained from the continuity equation at each port

$$Q_i = Q_{i+1} + q_i$$

The discharge q from the orifice (port) is obtained from $q_o = av_o$, where v_o is the velocity at the exit of the port obtained by expressing the energy equation across the orifice opening

$$q_o = av_o = aK_i \sqrt{2g \left(\frac{p_i}{\gamma} + \frac{v_i^2}{2g} + z_i - \frac{p_o}{\gamma} + z_o \right)} = aK_i \sqrt{2g \left(h_i + \frac{v_i^2}{2g} - h_o \right)}$$

where

$$K_i = C_d \sqrt{1 - \frac{V_{i+1}^2}{2g(H_i - H_0)}} \quad h = \frac{p}{\gamma} + z \quad H = h + \frac{v^2}{2g}$$

The head upstream is obtained by expressing the energy equation along the pipe segment

$$H_i = H_{i+1} + f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} \quad f = \frac{1}{4} \left[\log_{10} \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^{-2}$$

The solution in tabular form is as follows:

Port or Segment	q (m ³ /s)	Q (m ³ /s)	V (m/s)	H (m)	p/γ (m)	K	f	h_L (m)
3	0.06	0.06	0.477	11.53	11.52	0.6750	0.0212	0.0092
2	0.06	0.12	0.955	11.54	11.49	0.6744	0.0202	0.0353
1	0.06	0.18	1.432	11.57	11.47	0.6726	0.0199	2.599
Reservoir	0			14.17				

Therefore, the water surface elevation in the reservoir should be at least 14.2 m and the total discharge is 0.18 m³/s.

HomeWork IX

NAME:

ID #:

Design a wedge inlet transition structure ($\theta = 27.5^\circ$) to join a trapezoidal concrete-lined channel and a concrete flume of rectangular cross section. The channel is on a slope of 0.00035 and has side slopes of 1 vertical to 2 horizontal. The bottom width of the channel is 10 ft and the depth of flow is 3 ft. The invert elevation of the channel is 110 ft. Dimension the flume such that the depth is nearly equal to the width of the flume and the Froude number in the flume is around 0.50. The Manning's roughness coefficient is 0.015 and the inlet head loss is given by $kV^2/2g$ where $k = 0.2$ and V is the downstream velocity.

Solution

For uniform flow condition, Manning's equation gives

w (ft)	y (ft)	z (ft)	B (ft)	m	n	A (ft ²)	P (ft)	R (ft)	K (ft ³ /s)	Q (ft ³ /s)
113	3	110	10	2	0.015	48	23.42	2.05	7.694	143.9

The Froude number is given by

$$F_r = \frac{V}{\sqrt{gy}} = \frac{Q}{by\sqrt{gy}} = 0.5$$

Setting $b = y$, one obtains $y = 4.8$ ft for $Q = 144$ ft³/s. Selecting $b = 4.8$ ft and $y = 4.82$ ft, the length L based on the angle of 27.5° is

$$L = \frac{5 + 2(3) - 0.5(4.8)}{\tan(27.5^\circ)} = 16.5 \text{ ft}$$

The invert elevation is obtained from the energy equation using $k = 0.2$ for the head loss

$$y_1 + z_1 + \frac{V_1^2}{2g} = y_2 + z_2 + \frac{V_2^2}{2g} + \frac{k}{2g} V_2^2$$

Substituting the numerical values, one obtains the required flume invert z_2 .

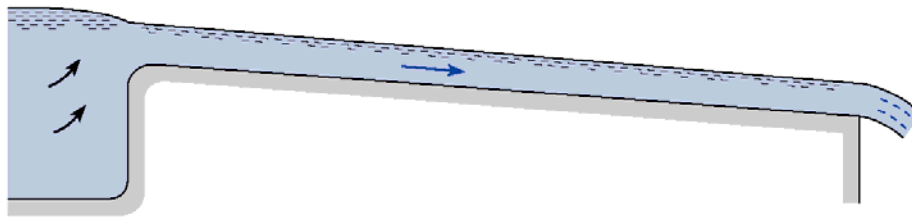
y_1 (ft)	A_1 (ft ²)	V_1 (ft/s)	z_1 (ft)	y_2 (ft)	A_2 (ft ²)	V_2 (ft/s)	z_2	h_e (ft)	ΔE (ft)
3	48	3	110	4.82	23.1	6.23	107.6	0.12	0.00

HomeWork X

NAME:

ID #:

The concrete rectangular channel shown is 3.5 m wide and has a bottom slope of 0.001. The reservoir water surface elevation is 2.5 m above the bed of the channel at the entrance. Assume Manning's n is 0.013 and neglect the head loss at the entrance. Whenever applicable, apply the energy equation on two or three reaches of the channel, i.e. at three or four sections.



1. Compute the discharge in the channel if it is 3000 m long
2. Compute the discharge in the channel if it is 100 m long
3. Compute the discharge in the channel if it is 100 m long with a slope of 0.01.

Solution

1. For a long channel with a mild slope and discharging with a free outfall, the depth upstream reaches the normal depth. Expressing the energy equation between the reservoir surface and the channel entrance, one gets

$$2.5 = y + \frac{V^2}{2g}$$

The velocity in the channel V is given by Manning's equation for normal depth $V = R^{2/3} S^{1/2} / n$. The hydraulic radius in terms of the depth y is given by $R = 3.5y_n / (2y_n + 3.5)$. Substituting in the energy equation, one can solve for y . The depth is $y_n = 2.21$ m and the corresponding flow rate is $Q = 18.5$ m³/s.

2. For a short channel, uniform flow will not become established in the upstream part of the channel. One must therefore solve the energy equation between the downstream depth and the upstream reservoir. A flow rate is assumed, the critical depth downstream is calculated and the water surface profile is then computed. Iterate on Q until the upstream depth is equal to the given depth (one can also iterate on the critical depth, solve for Q and calculate the water surface profile). For two sections, the iterated flow rate is $Q = 28$ m³/s. The intermediate calculations using the standard step method are

Sec	x	z	Depth	Area	V	R	K	Mean K	S	hl	H
1	0	0.00	1.87	6.54	4.28	0.90	470.22				2.80
2	50	0.05	2.24	7.84	3.57	0.98	596.43	533.33	0.00276	0.13782	2.94
3	100	0.10	2.51	8.80	3.18	1.03	690.95	643.69	0.00189	0.09461	3.04

Note that $K = AR^{2/3} / n$

3. For a short and steep channel, the depth at the entrance is critical. Expressing the energy equation between the reservoir and the critical section and using the critical depth equation, one obtains $Q = 23.57$ m³/s. One must check for steepness by comparing with normal depth.

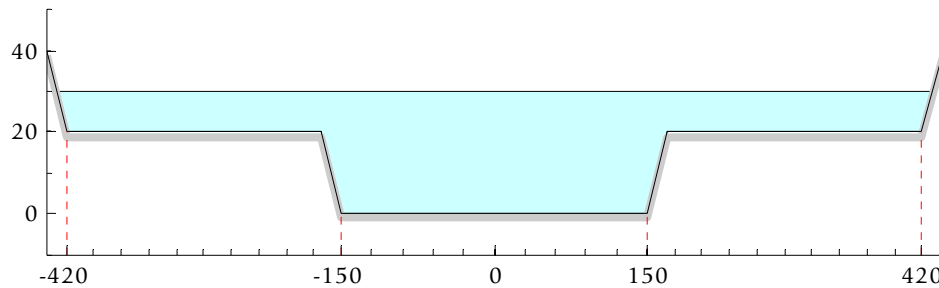
Homework Problems 2008

HomeWork I

NAME:

ID #:

Compute the minimum height of the levees necessary to protect from the 50-yr flood of magnitude of 172,000 cfs if it is to have 2 ft of freeboard. The main channel is covered with gravel beds ($n=0.035$) while the overbank areas are covered with tall grass ($n=0.040$). The uniform slope of the channel is 0.15%. The channel cross-section with side slopes of 1:1 is shown below



Solution

- The area of the main channel is $A_c = 300(20) + 20^2 + 340y$ where y is the height above the overbank level
- The wetted perimeter for the channel is $P_c = 300 + 2(20)\sqrt{2} = 356.57$.
- The area of each overbank is $A_b = 250y + y^2/2$ and
- The wetted perimeter is $P_b = 250 + y\sqrt{2}$.

The total discharge for various depths is given below

y	A_c	P_c	R_c	Q_c	A_b	P_b	R_b	Q_b	Q_t
n				0.035				0.040	$10^3 \text{ ft}^3/\text{s}$
0	6400	356.57	17.95	72.34	0.0	0.00	0.00	0.00	72.34
1	6740	356.57	18.90	78.86	250.5	251.41	1.00	0.36	79.58
2	7080	356.57	19.86	85.60	502.0	252.83	1.99	1.14	87.88
3	7420	356.57	20.81	92.56	754.5	254.24	2.97	2.25	97.05
4	7760	356.57	21.76	99.73	1008.0	255.66	3.94	3.63	106.99
5	8100	356.57	22.72	107.12	1262.5	257.07	4.91	5.26	117.65
6	8440	356.57	23.67	114.72	1518.0	258.49	5.87	7.13	128.98
7	8780	356.57	24.62	122.53	1774.5	259.90	6.83	9.21	140.95
8	9120	356.57	25.58	130.53	2032.0	261.31	7.78	11.51	153.55
9	9460	356.57	26.53	138.75	2290.5	262.73	8.72	14.00	166.74
10	9800	356.57	27.48	147.16	2550.0	264.14	9.65	16.68	180.52
9.39	9591	356.57	26.90	141.98	2390.7	263.27	9.08	15.01	172.00

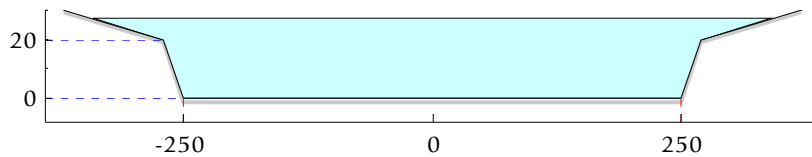
The minimum height of the levees is therefore 11.39 ft or 12 ft above the overbank level.

HomeWork II

NAME:

ID #:

Determine the width of the flood plain for a 100-yr summer flood of 110,000 cfs for the cross section shown. The channel has a uniform slope of 0.09% with a gravel bed ($n = 0.035$) and side slopes of 1:1, while the overbank area is brush-covered with some trees ($n = 0.06$) and side slopes of 1:10 (V:H).



Solution

- The depth for a flow of 110,000 cfs is obtained by applying Manning's equation for the compounded section.
- The area of the main channel is $A_c = 500(20) + 20^2 + 540y$ where y is the height above the overbank level (i.e. El. 20)
- The wetted perimeter is $P_c = 500 + 2(20)\sqrt{2} = 556.57$.
- The area of each overbank is $A_b = 5y^2$ and the wetted perimeter is $P_b = y\sqrt{101}$.
- The discharge for the main channel is given by

$$Q_c = \frac{1.49}{0.035} A_c \left(\frac{A_c}{P_c} \right)^{2/3} \sqrt{0.0009}$$

- The discharge for each overbank is

$$Q_b = \frac{1.49}{0.06} A_b \left(\frac{A_b}{P_b} \right)^{2/3} \sqrt{0.0009}$$

- The total discharge for selected depths $y + 20$ is given in the table below

Depth (ft)	20	25	21	22
Flow (1000 cfs)	93.53	137.75	101.77	110.00

- The depth corresponding to a discharge of 110,000 cfs is therefore around 22 ft
- The width of the flood plain is therefore $10(y - 20) = 20$ ft on each side

HomeWork III

NAME:

ID #:

Water flows uniformly in a 2 m wide rectangular channel at a rate of $1.6 \text{ m}^3/\text{s}$ and a depth of 0.75 m. Calculate the change in water surface elevation at a section contracted to a 1.4 m width with an 8 cm depression in the bottom.

Solution

- The depth is greater than the critical depth of 0.4 m. Hence the flow is subcritical.
- Expressing the energy equation between section 1 and 2 at the depression, one gets

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g}$$

- Here $z_1 = 0$, $z_2 = -0.08$, $y_1 = 0.75$, and $V_1 = 1.067 \text{ m/s}$.
- The velocity at section 2 is given by $V_2 = 1.6/(1.4y_2)$ where y_2 is the unknown depth
- One can then solve for the depth y_2 by successive iterations to get 0.778 m
- Note that the second root is the sequent depth for supercritical depth and is disregarded.
- The change in water surface elevation $w_1 - w_2$ is therefore $y_1 + z_1 - y_2 - z_2 = 0.052 \text{ m}$

HomeWork IV

NAME:

ID #:

A rectangular concrete channel with $n = 0.02$ changes from a mild slope to a steep slope. The prismatic channel is 20 m wide and the rate of flow is $180 \text{ m}^3/\text{s}$. The slope of the mild portion of the channel is 0.0006 m/m . Calculate the distance from the slope breakpoint to the point where the depth is 3 m using a maximum depth increment of 0.2 m.

Solution

Because of the change in slope, the flow switches from a subcritical to a supercritical flow conditions at the slope breakpoint. Upstream of the slope breakpoint, the water surface profile is M2 and downstream of the slope, it is S2. The water surface profile is calculated using the direct step method since the channel configuration is uniform.

The length of the reach Δx is solved using the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \quad E = y + \frac{V^2}{2g}$$

and the friction slope is either given by

$$S_f = \frac{n^2 V^2}{R^{4/3}} \quad \text{or} \quad S_f = \frac{Q^2}{K^2}$$

where $K = AR^{2/3}/n$. The critical depth is $y_c = \sqrt[3]{9^2/9.81} = 2.02$.

The results for both formulations are tabulated below

Section	y	R	V	Mean R	Mean V	E	S	Δx	x
							10^3		
1	2.02	1.68	4.45			3.03			0.00
2	2.20	1.80	4.09	1.74	4.27	3.05	3.48	-7.36	-7.36
3	2.40	1.94	3.75	1.87	3.92	3.12	2.67	-30.81	-38.17
4	2.60	2.06	3.46	2.00	3.61	3.21	2.06	-64.16	-102.33
5	2.80	2.19	3.21	2.13	3.34	3.33	1.63	-112.41	-214.74
6	3.00	2.31	3.00	2.25	3.11	3.46	1.31	-185.66	-400.40

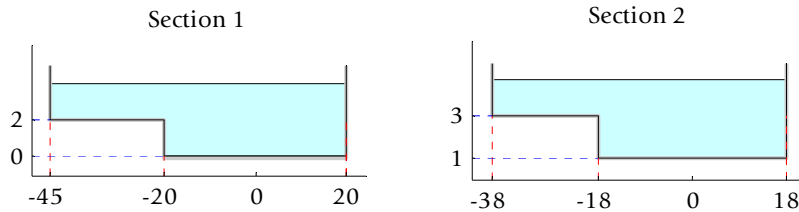
Section	y	R	V	K	Mean K	E	S	Δx	x
							10^3		
1	2.02	1.68	4.45	2857.88	0	3.03	0	0	0
2	2.2	1.8	4.09	3259.35	3058.62	3.05	3.46	-7.41	-7.41
3	2.4	1.94	3.75	3727.39	3493.37	3.12	2.65	-31.03	-38.44
4	2.6	2.06	3.46	4214.14	3970.76	3.21	2.05	-64.59	-103.02
5	2.8	2.19	3.21	4718.35	4466.24	3.33	1.62	-113.12	-216.15
6	3	2.31	3	5238.9	4978.62	3.46	1.31	-186.85	-402.99

HomeWork V

NAME:

ID #:

Compute the water surface profile at the upstream section 2 given that the water surface elevation at section 1 is 3 m. The river is subject to a flood of 285 m³/s. The distance between the two sections is 400 m and Manning's *n* is 0.05 in the overbank areas and 0.03 in the main channel. Use expansion and contraction coefficients of 0.3 and 0.1, respectively.



Solution

The water surface profile is computed from the energy equation in the following form

$$y_1 + \frac{\alpha_1 V_1^2}{2g} + z_1 = y_2 + \frac{\alpha_2 V_2^2}{2g} + z_2 + S_f \Delta x + C_l \left| \frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right|$$

where $S_f = Q^2 / \bar{K}^2$, $K = AR^{2/3} / n$, $C_l = C_e$ is the expansion coefficient, and

$$\alpha = \frac{A_T^2}{K_T^3} \sum_{i=1}^N \left(\frac{K_i^3}{A_i^2} \right)$$

The detailed solution is tabulated below

Sec	Try	<i>x</i>	<i>w</i>	<i>y</i>	<i>z</i>	<i>B</i>	<i>m</i>	<i>n</i>	<i>A</i>	<i>P</i>	<i>R</i>	<i>K</i>
												10 ³
1		0	3	1	2	25	0	0.05	25	26	0.9615	0.4871
				3	0	40	0	0.03	120	45	2.667	7.692
2	1	-400	4	1	3	20	0	0.05	20	21	0.9524	0.3872
				3	1	36	0	0.03	108	41	2.634	6.866
2	2	-400	3.2	0.2	3	20	0	0.05	4	20.2	0.198	0.0272
				2.2	1	36	0	0.03	79.2	40.2	1.97	4.149
2	3	-400	3.5	0.5	3	20	0	0.05	10	20.5	0.4878	0.1239
				2.5	1	36	0	0.03	90	40.5	2.222	5.109
2	4	-400	3.55	0.55	3	20	0	0.05	11	20.55	0.5353	0.145
				2.55	1	36	0	0.03	91.8	40.55	2.264	5.276

Sec	<i>w</i>	<i>A_T</i>	<i>K_T</i>	α	<i>V</i>	KE	<i>Q_m</i>	<i>K_m</i>	<i>S_f</i>	<i>h_L</i>	<i>h_e</i>	ΔE
			10 ³					10 ³	10 ⁻⁶	10 ⁻³	10 ⁻³	
1		145	8.179	1.222	1.966	0.2405	0	0	0	0	0	0
2	4	128	7.254	1.198	2.227	0.3026	285	7.716	1.364	-545.7	-18.64	-0.4978
2	3.2	83.2	4.176	1.082	3.425	0.6473	285	6.178	2.128	-851.4	-122	0.3666
2	3.5	100	5.233	1.15	2.85	0.4762	285	6.706	1.806	-722.5	-70.7	0.0575
2	3.55	102.8	5.421	1.158	2.772	0.4535	285	6.8	1.757	-702.6	-63.9	0.0036

HomeWork VI

NAME:

ID #:

Design a cylinder quadrant inlet transition structure connecting an earth canal having a bottom width of 18 ft and side slopes of 2H:1V to a rectangular concrete flume 12 ft 6 in wide. The design discharge is 314.5 cfs and the Froude number in the flume should be around 0.50. The invert elevation of the channel is 60 ft and it is on a slope of 0.0005. Manning's roughness coefficient is equal to 0.014 in the flume and to 0.0255 in the canal. The inlet head loss is given by $kV^2/2g$ where $k = 0.15$ and V is the downstream velocity.

Solution

For uniform flow condition, Manning's equation gives a normal depth of 4.3 ft.

The Froude number is given by

$$F_r = \frac{V}{\sqrt{gy}} = \frac{Q}{by\sqrt{gy}} = 0.5$$

Setting $b = 12.5$ ft, one obtains $y = 4.3$ ft for $Q = 314.5$ ft³/s. The length L is the radius

$$L = \frac{9 + 2(4.3) - 0.5(12.5)}{\tan(45^\circ)} = 11.35 \text{ ft}$$

The invert elevation is obtained from the energy equation using $k = 0.2$ for the head loss

$$y_1 + z_1 + \frac{V_1^2}{2g} = y_2 + z_2 + \frac{V_2^2}{2g} + \frac{k}{2g} V_2^2$$

Substituting the numerical values, one obtains the required flume invert z_2 .

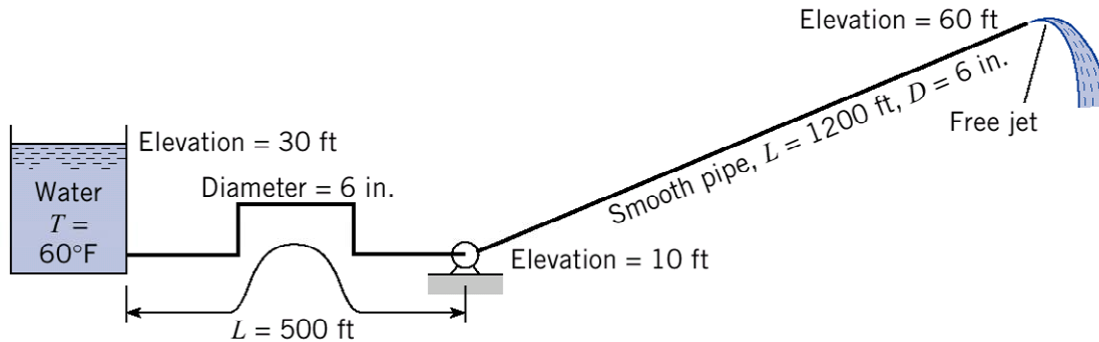
Q (cfs)	y_1 (ft)	A_1 (ft ²)	V_1 (ft/s)	KE_1 (ft)	z_1 (ft)	y_2 (ft)	A_2 (ft ²)	V_2 (ft/s)	KE_2 (ft)	h_e (ft)	z_2 (ft)	ΔE (ft)
314.5	4.3	114.2	2.75	0.118	60	4.3	53.5	5.87	0.535	0.08	59.5	0.00

HomeWork VII

NAME:

ID #:

Compute the horsepower (1 HP = 550 lb-ft/s) needed to pump water at a rate of 2 cfs if the pump efficiency is 80%. Calculate also the pressure in lb/in² at the midpoint of the long pipe. The four bends have a radius of 12 in. Assume smooth pipe ($k_s = 0$), $\gamma = 62.4$ lb/ft³ & $\nu = 1.22 \times 10^{-5}$ ft²/s.



Solution

Expressing the energy equation between the lower reservoir and the jet outlet, one obtains

$$0 + 0 + 30 + h_p = 0 + \frac{V_2^2}{2g} + 60 + \left(k_e + 4k_b + f_1 \frac{L_1}{D_1} + f_2 \frac{L_2}{D_2} \right) \frac{V_2^2}{2g}$$

The discharge rate is given as 2 cfs. Hence, the velocity is 10.19 cfs in both pipes and the velocity head is 1.612 ft. The friction factor is given by

$$f = 0.25 \left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^{-2} = 0.0135$$

where $k_s = 0$ for smooth pipe and $Re = 417500$. The minor loss coefficients are $k_e = 0.5$ for entrance and $k_b = 0.19$ for 90° bend with $r/D = 2$. Therefore, the sum of head losses is $47.16 V_2^2 / 2g = 76$ ft.

Hence, the pump head is $h_p = 107.7$ ft and the pump horsepower for 80% efficiency is

$$P = \frac{\gamma Q h_p}{0.8(550)} = 30.5 \text{ HP}$$

The pressure at midpoint is given by the energy equation expressed between the midpoint and the free jet

$$\frac{p_m}{\gamma} + \frac{V_2^2}{2g} + 35 = 0 + \frac{V_2^2}{2g} + 60 + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

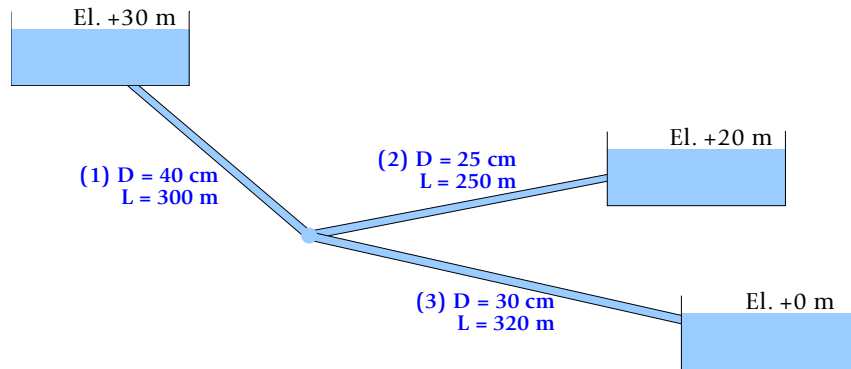
Hence, the pressure head $p_m / \gamma = 51.1$ ft and the pressure is $p_m = 3191$ lb/ft² or 22.16 psig.

HomeWork VIII

NAME:

ID #:

Determine the flow rate into or out of each reservoir. Assume $f = 0.025$.



Solution

Expressing the energy equation between reservoir 1 and the junction, one obtains

$$30 = H_j + \left(0.5 + 0.025 \frac{300}{0.4} \right) \frac{V_1^2}{2g}$$

Expressing the energy equation between the junction and reservoir 3, one obtains

$$H_j = \left(1 + 0.025 \frac{320}{0.3} \right) \frac{V_3^2}{2g}$$

Assuming that $H_j = 20$, one obtains $V_1 = 3.23$ m/s and $V_3 = 3.83$ m/s. The flow rates are therefore $Q_1 = 0.40$ m³/s and $Q_3 = 0.27$ m³/s. Hence, the flow is going into reservoir 2 and the head at the junction must be greater than 20 to account for it. The third reservoir equation is

$$H_j = 20 + \left(1 + 0.025 \frac{250}{0.25} \right) \frac{V_2^2}{2g}$$

Finally, the continuity equation at the junction is

$$Q_1 = Q_2 + Q_3$$

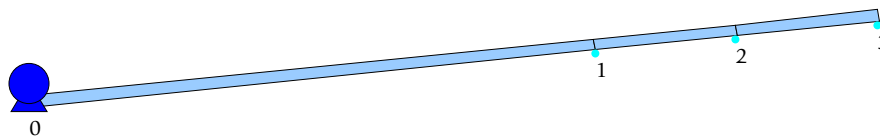
Substituting the velocity terms in the continuity equation ($Q = VA$) and solving for the unknown head at the junction H_j by iteration, one obtains $H_j = 22.44$ m. The flow rates are therefore $Q_1 = 0.348$ m³/s, $Q_2 = 0.066$ m³/s and $Q_3 = 0.282$ m³/s.

HomeWork IX

NAME:

ID #:

Compute the minimum pump power required to supply an irrigation pipe system composed of a supply line and three port outlets laid on an uphill slope of 10% as shown. The discharge rate at the last port in the pipe manifold system is 0.8 l/s. There are 3 discharge openings that are 50 m spaced apart and the distance from the pump to the first opening is 200 m. The diameter of the manifold pipe is 5 cm and the diameter of the ports is 1 cm. Use a constant friction factor $f = 0.025$ and a constant orifice discharge coefficient $C_D = 0.675$ for the ports.



Solution

The flow rates in the manifold system is obtained from the continuity equation at each port

$$Q_i = Q_{i+1} + q_i$$

The discharge q from the orifice (port) is obtained from $q_o = av_o$, where v_o is the velocity at the exit of the port obtained by expressing the energy equation between two points across the opening

$$q_o = av_o = aC_D \sqrt{2g \left(\frac{p_i}{\gamma} + \frac{v_i^2}{2g} + z_i - \frac{p_o}{\gamma} + z_o \right)} = aC_D \sqrt{2g \left(h_i + \frac{v_i^2}{2g} - h_o \right)}$$

The head upstream is obtained by expressing the energy equation along the pipe segment

$$H_i = H_{i+1} + f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} \quad H = \frac{p}{\gamma} + \frac{v^2}{2g} + z$$

The solution in tabular form is as follows:

Port or Segment	x (m)	z (m)	q (m ³ /s)	Q (m ³ /s)	H (m)	V (m/s)	p/γ (m)	h_L (m)
3	300	30	0.00080	0.00080	41.6	0.407	11.60	0.2115
2	250	25	0.00096	0.0018	41.82	0.898	16.78	1.027
1	200	20	0.00111	0.0029	42.84	1.469	22.74	11.0
Pump	0	0			53.85			

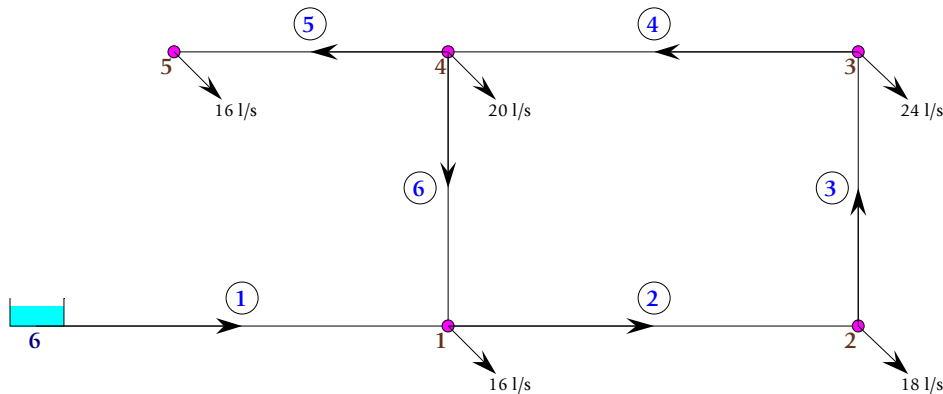
Therefore, the pump power required is $P = \gamma QH = 9.81(0.0029)53.85 = 1.52$ kW

HomeWork X

NAME:

ID #:

Write down the system of equations for the given network expressing all coefficients in numerical form. Compute also the pressure at junction 1. Assume $f=0.018$



The hydraulic data are:

Junction	1	2	3	4	5	6	Pipe	1	2	3	4	5	6
Elevation (m)	55	48	58	45	44	100	Length (m)	1500	300	250	275	245	225
Demand (l/s)	16	18	24	20	16		Diameter (cm)	25	15	15	15	15	20

Solution

There are six unknown pipe flow rates. The six equations are the five continuity equations and the energy equation around the loop. The continuity equations are

$$\text{J-1: } Q_1 + Q_6 - Q_2 = 0.016$$

$$\text{J-2: } Q_2 - Q_3 = 0.018$$

$$\text{J-3: } Q_3 - Q_4 = 0.024$$

$$\text{J-4: } Q_4 - Q_5 - Q_6 = 0.02$$

$$\text{J-5: } Q_5 = 0.016$$

$$\text{Loop 2346: } k_2 Q_2^x + k_3 Q_3^x + k_4 Q_4^x + k_6 Q_6^x = 0$$

$$\text{where } k = 8fL / g\pi^2 D^5$$

Pipe	1	2	3	4	5	6
$k \text{ (s}^2/\text{m}^5) \times 10^3$	2.285	5.876	4.896	5.386	4.798	1.046

The flow rate in pipe 1 is the sum of all demands, i.e. 94 l/s. The velocity is then 1.915 m/s. The head at junction 1 can be computed from the energy equation

$$H_1 = H_6 - f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = 100 + 0.018 \frac{1500}{0.25} \left(\frac{4 \cdot 0.094}{\pi \cdot 0.25^2} \right)^2 \frac{1}{2(9.81)} = 80 \text{ m}$$

The pressure head is

$$\frac{P_1}{\gamma} = H_1 - \frac{V_1^2}{2g} - z_1 = 80 - \frac{1.915^2}{2(9.81)} - 55 = 24.8$$

The pressure is therefore $P_1=243.3 \text{ kPa}$.

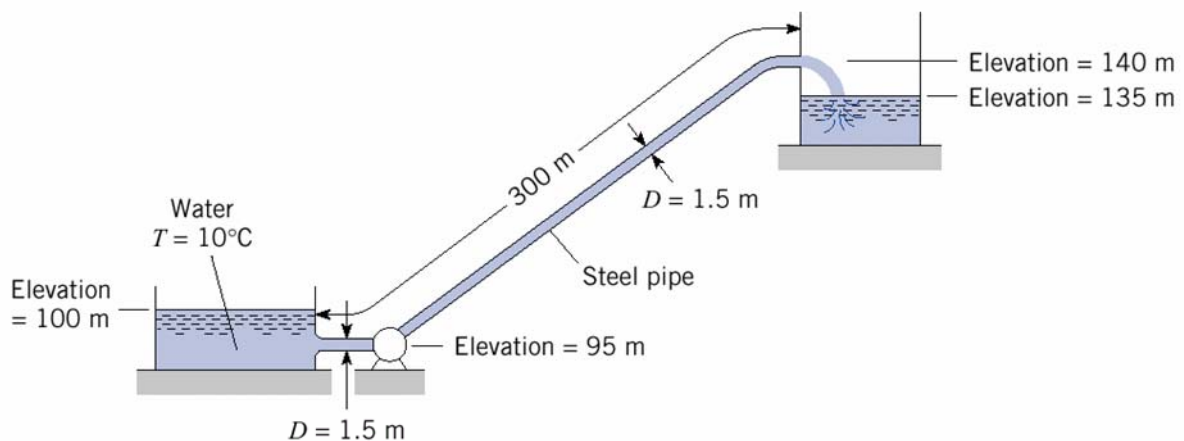
Homework Problems 2009

HomeWork I

NAME:

ID #:

Compute the flow rate in the system shown below if the pump power is 8000 kW. The pump efficiency is 90%, the equivalent sand-grain roughness of the steel pipe is $k_s = 0.046$ mm and the kinematic viscosity of water is 1.31×10^{-6} m²/s.



Solution

Expressing the energy equation between the lower reservoir and the pipe outlet, one obtains

$$0 + 0 + 100 + h_p = 0 + \frac{V^2}{2g} + 140 + \left(k_e + f \frac{L}{D} \right) \frac{V^2}{2g}$$

The pump power is given by

$$P = \frac{\gamma Q h_p}{0.9} = 8000 \text{ or } h_p = \frac{734}{Q}$$

Using a minor loss coefficient of $k_e = 0.12$ for rounded entrance, the energy equation in terms of Q becomes

$$\frac{734}{Q} = 40 + \left(1 + 0.12 + f \frac{300}{1.5} \right) \frac{1}{2g} \frac{Q^2}{(1.767)^2}$$

The friction factor is also given by

$$f = 0.25 \left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^{-2}$$

where $k_s = 0.046 \times 10^{-3}$ m. Solving for Q using an initial estimate of $f = 0.02$, one gets $Q = 13.36$ m³/s. Computing the new value for f and solving for Q in an iterative fashion, one obtains after two to three iterations that $f = 0.0102$ and $Q = 14.45$ m³/s.

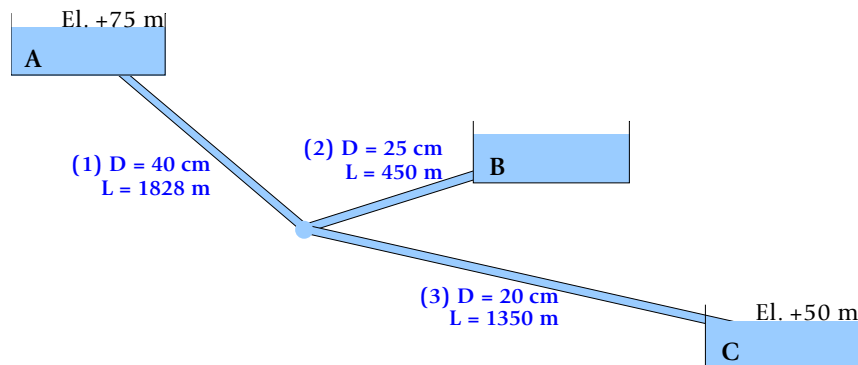
Common Errors: El. 135 used; k.exit used; wrong pump power eq; wrong f calculation;

HomeWork II

NAME:

ID #:

Determine the flow rate into or out of reservoir A and C given that the discharge into reservoir B is 90 l/s. Assume $f = 0.018$ and neglect all minor losses.



Solution

Expressing the energy equation between reservoir A and the junction, one obtains

$$75 = H_j + \left(0.018 \frac{1828}{0.4} \right) \frac{V_1^2}{2g}$$

Expressing the energy equation between reservoir C and the junction, one obtains

$$H_j = 50 + \left(0.018 \frac{1350}{0.2} \right) \frac{V_3^2}{2g}$$

The continuity equation at the junction is

$$Q_1 = Q_2 + Q_3 = 0.09 + Q_3$$

Substituting the velocity terms in the continuity equation ($Q = VA$)

$$\frac{\pi(0.4)^2}{4} \sqrt{\frac{75 - H_j}{4.193}} - \frac{\pi(0.2)^2}{4} \sqrt{\frac{H_j - 50}{6.193}} - 0.09 = 0$$

Solving for the unknown head at the junction H_j by iteration, one obtains $H_j = 69.37$ m.

The flow rates are therefore $Q_1 = 0.1457$ m³/s, $Q_2 = 0.09$ m/s and $Q_3 = 0.0557$ m/s.

The water surface elevation in Reservoir B is then 63.8 m. (Question not asked)

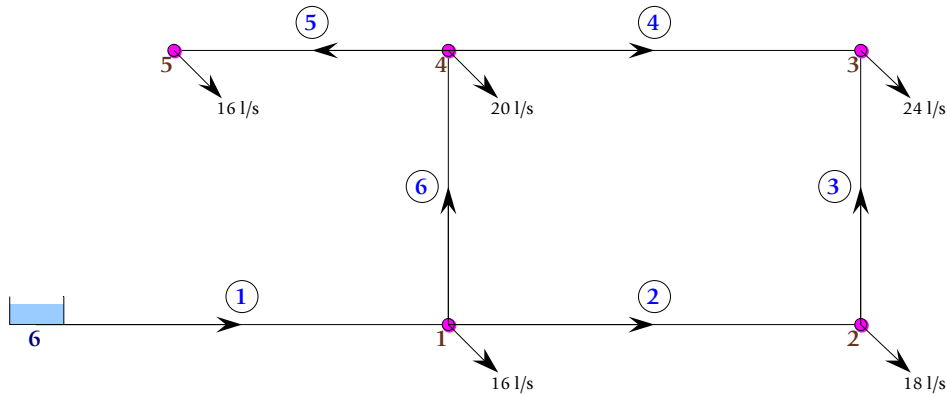
Common Errors: missing area term; insufficient iteration so that continuity is not conserved.

HomeWork III

NAME:

ID #:

Compute the flow rates in the system. The hydraulic data are shown below. Assume $f=0.018$.



Junction	1	2	3	4	5	6	Pipe	1	2	3	4	5	6
Elevation (m)	55	48	58	45	44	100	Length (m)	1500	300	250	275	245	225
Demand (l/s)	16	18	24	20	16		Diameter (cm)	25	15	15	15	15	20

Solution

There are 6 unknown pipe flow rates and 6 equations (5 continuity equations and the energy equation around the loop). The equations are

$$\text{J-1: } Q_1 = Q_2 + Q_6 + 0.016$$

$$\text{J-2: } Q_2 = Q_3 + 0.018$$

$$\text{J-3: } Q_4 + Q_3 = 0.024$$

$$\text{J-4: } Q_6 = Q_4 + Q_5 + 0.02$$

$$\text{J-5: } Q_5 = 0.016$$

$$\text{Loop 2346: } k_2 Q_2^2 + k_3 Q_3^2 - k_4 Q_4^2 - k_6 Q_6^2 = 0$$

Here $k = 8fL/g\pi^2 D^5$

Pipe	1	2	3	4	5	6
$k \text{ (s}^2/\text{m}^5) \times 10^3$	2.285	5.876	4.896	5.386	4.798	1.046

Expressing the loop equation in terms of Q_4 using the continuity equations, one gets

$$k_2 (0.018 + 0.024 - Q_4)^2 + k_3 (0.024 - Q_4)^2 - k_4 Q_4^2 - k_6 (Q_4 + 0.016 + 0.02)^2 = 0$$

Iterating on Q_4 , one obtains $Q_4 = 16.1$ l/s. From the above equations, the flow rates are: $Q_3 = 7.9$ l/s, $Q_2 = 25.9$ l/s, and $Q_6 = 52.1$ l/s. $Q_1 = 94$ l/s is the sum of all demands as expected.

Note: One should express Q^2 as $Q|Q|$ to ensure the correct flow direction in the loop equation. The above loop equation gives the correct solution because the flow directions happen to be correct. The loop equation should have been expressed as

$$k_2 |Q_2| Q_2 + k_3 |Q_3| Q_3 - k_4 |Q_4| Q_4 - k_6 |Q_6| Q_6 = 0$$

Common Errors: inconsistent units; sign in loop equation; $H_1 = 55$ m; missing demands in continuity equations.

HomeWork IV

NAME:

ID #:

Solve HomeWork III problem using EPANET software.

Solution

The EPANET software does not accept the friction factor as an input. The roughness size must be provided upon which the friction factor is calculated. The value of k_s should be around 0.04 mm.

HomeWork V

NAME:

ID #:

A circular concrete culvert ($n = 0.012$) is to carry a discharge of $50 \text{ ft}^3/\text{s}$ on a slope of 0.0010 . Determine the smallest suitable culvert diameter if it is to flow not more than $\frac{3}{4}$ full. The available culvert pipes diameters from the manufacturer are multiples of 1 ft.

Solution

The discharge for a circular culvert is given by

$$Q_c = \frac{1.49}{0.012} A_c \left(\frac{A_c}{P_c} \right)^{2/3} \sqrt{0.0010} = 50$$

The area of a circular channel partially full is given by

$$A = \frac{D^2}{8} (\theta - \sin \theta) = \frac{r^2}{2} (\theta - \sin \theta)$$

where θ is the lower arc angle. The corresponding area for a $\frac{3}{4}$ full flow is

$$\frac{D^2}{8} (\theta - \sin \theta) = \frac{3}{4} \frac{\pi D^2}{4}$$

The corresponding angle is then the solution of

$$\theta - \sin \theta = \frac{4\pi}{3}$$

That is $\theta = 3.97$ or 228° , or equivalently, the upper half-angle is 66° . The wetted perimeter is $P = \theta D/2$ and the hydraulic radius is

$$R_h = \frac{D}{4\theta} (\theta - \sin \theta)$$

The culvert diameter is the solution of

$$\frac{1.49}{0.012} \frac{D^2}{8} (\theta - \sin \theta) \left[\frac{D}{4\theta} (\theta - \sin \theta) \right]^{2/3} \sqrt{0.0010} = 50$$

Iterating on D , one obtains $D = 4.29$ ft. Hence, the smallest suitable culvert diameter is 5 ft.

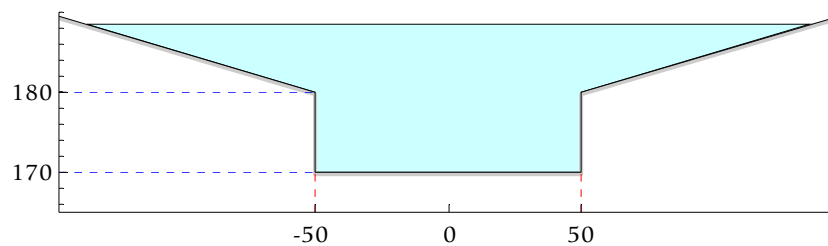
Common Errors: Assumption of $\frac{3}{4}$ P or $\frac{3}{4}$ y with $\frac{3}{4}$ A; wrong derivation of A (although available in formula sheet)

HomeWork VI

NAME:

ID #:

Determine the width of the floodway for the following river cross-section that carries a 50-yr peak flow of 5,000 cfs given that the river can carry a flow of 3,400 cfs with a water surface elevation of 185 ft. Compute also the maximum allowable encroachment for an increase in depth not to exceed ½ ft for the 50-yr peak flow. The side slopes of the overbank sections are 1:10 (V:H). Assume Manning's n is 0.06 for the overbank areas and 0.035 for the main channel.



Solution

- The discharge for the main channel and overbanks are given by Manning's equation

$$Q_c = \frac{1.49}{0.035} A_c \left(\frac{A_c}{P_c} \right)^{2/3} \sqrt{S} \qquad Q_b = \frac{1.49}{0.06} A_b \left(\frac{A_b}{P_b} \right)^{2/3} \sqrt{S}$$

- The depth for a flow of 5000 cfs must first be obtained.
- To find the depth for $Q = 5000$ cfs, one needs the slope of the channel S .
- The area of the main channel is $A_c = 100(10) + 100y$ where y is the height above the overbank level (i.e. El. 180). The wetted perimeter is $P = 120$ ft.
- The area of each overbank is $A_b = 5y^2$ and the wetted perimeter is $P_b = y\sqrt{101}$.
- S is obtained from Manning's equation with $Q = 3400$ cfs and $y = 185'$ and is $= 0.0000915$
- The depth for $Q = 5000$ cfs is $y = 188.3$ ft after few iteration trials as shown below

Depth y (ft)	16	17	18	18.3
Flow (cfs)	3840	4320	4840	5005

- The width of the floodway is then 83 ft on each side.
- For a maximum allowable increase in depth of 0.5 ft, the new depth is 188.8 ft.
- The area of the main channel is 1880 ft² and the hydraulic radius is 15.66 ft.
- The width of the encroachment is b and the available flow area is $5y^2 - b^2/20$ ft on each side. The wetted perimeter is $P_b = (y - b/10)\sqrt{101} + b/10$.
- Solving Manning's equation for b with $Q = 5000$ and $y = 188.8$ ft, one gets $b = 70.5$ ft.

HomeWork VII

NAME:

ID #:

Water flows uniformly in a 2 m wide rectangular channel at a rate of $1.6 \text{ m}^3/\text{s}$ and a depth of 0.75 m. Calculate the change in water surface elevation at a section contracted to a 1 m width with an 8 cm depression in the bottom.

Solution

- The critical depth at section 1 and 2 are 0.403 and 0.639, respectively
- The actual depth is greater than the critical depth and the flow is thence subcritical.
- Expressing the energy equation between section 1 and 2 at the depression, one gets

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g}$$

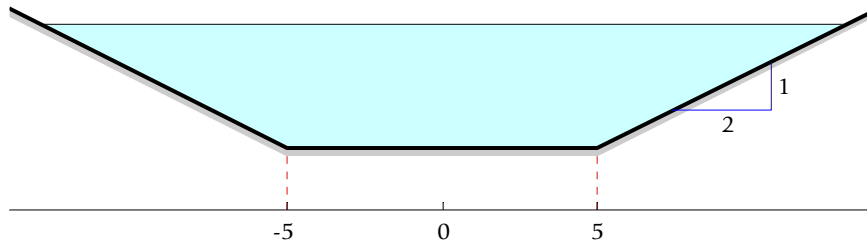
- Here $z_1 = 0$, $z_2 = -0.08$, $y_1 = 0.75$, and $V_1 = 1.067 \text{ m/s}$.
- The velocity at section 2 is given by $V_2 = 1.6/(1.4y_2)$ where y_2 is the unknown depth
- One can then try to solve for the depth y_2 by successive iterations but no solution is found
- The depth at section 2 is therefore the critical depth. Hence, $y_2 = 0.639 \text{ m}$
- The water surface elevation at section 2 is then $0.639 - 0.08 = 0.559 \text{ m}$
- At section 1, there is a damming effect and the water surface increases.
- Using the energy equation with $V_2 = 2.504 \text{ m/s}$, one gets after few iterations $y_2 = 0.8314 \text{ m}$.
- The change in water surface elevation $w_1 - w_2$ is therefore $0.8314 - 0.559 = 0.27 \text{ m}$

HomeWork VIII

NAME:

ID #:

A trapezoidal channel with the dimensions as shown is laid on a slope of 0.001 ft/ft. The Manning's n value for this channel is 0.025 and the rate of flow is 1000 cfs. Compute the water surface profile from the free outfall section to the point where $y \leq 0.9y_n$. Use increments of $\frac{1}{2}$ foot.



Solution

The critical depth at the free outfall section is obtained from

$$\frac{TQ^2}{gA^3} = \frac{(1000)^2 (10 + 4y)}{32.2 (10y + 2y^2)^3} = 1$$

Solving for y , one gets $y = 4.9$ ft. From Manning's equation, the uniform flow depth is 7.74 ft. The length of the reach Δx is obtained from the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \quad E = y + \alpha \frac{V^2}{2g} \quad S_f = \frac{Q^2}{K^2} \quad K = \frac{1.49}{n} AR^{2/3}$$

The water surface profile is calculated using the direct step method from 4.9 ft to 6.9 ft ($0.9y_n$) in increments of 0.5 ft.

Section	Depth	A	P	R	$K (10^4)$	V	E	$K_m (10^4)$	$S_f (10^3)$	Δx	x
1	4.9	97.02	31.91	3.04	1.213	10.31	6.55	0	0	0	0
3	5.4	112.3	34.15	3.289	1.481	8.903	6.631	1.347	5.511	-18	-18
4	5.9	128.6	36.39	3.535	1.779	7.775	6.839	1.63	3.765	-75.15	-93.14
5	6.4	145.9	38.62	3.778	2.11	6.853	7.129	1.944	2.645	-176.6	-269.8
6	6.9	164.2	40.86	4.019	2.474	6.089	7.476	2.292	1.904	-383.5	-653.2

**American University of Beirut
Department of Civil & Environmental Engineering**

**CIVE 440 Hydraulics & Laboratory
Due Date: 08-Jan-10**

Fall 2009

HomeWork IX

NAME:

ID #:

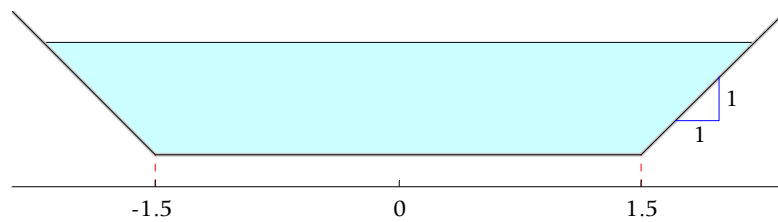
Solve HomeWork VIII problem using HEC-RAS software

HomeWork X

NAME:

ID #:

A trapezoidal channel with the dimensions as shown is laid on a slope of 0.001. The Manning's n value for this channel is 0.025 and the rate of flow is $20 \text{ m}^3/\text{s}$. Compute the water surface profile at an upstream distance of 10 m from the free outfall section. Use expansion and contraction coefficients of 0.3 and 0.1, respectively, and carry out two iterations only starting with $y = 1.6 \text{ m}$.



Solution

The water surface profile is computed from the energy equation in the following form

$$y_u + \frac{\alpha_u V_u^2}{2g} + z_u = y_d + \frac{\alpha_d V_d^2}{2g} + z_d + S_f \Delta x + C_l \left| \frac{\alpha_d V_d^2}{2g} - \frac{\alpha_u V_u^2}{2g} \right|$$

where $S_f = Q^2 / \bar{K}^2$ and $K = AR^{2/3} / n$. The detailed solution is tabulated below

Sec	Iter	x	w	z	y	B	m	n	A	P	R	K
												10^3
1		0	1.405	0	1.405	3	1	0.025	6.187	6.973	0.8873	0.2285
2	1	-10	1.61	0.01	1.6	3	1	0.025	7.36	7.525	0.978	0.2901
4	-10	1.81	0.01	1.8	3	1	0.025	8.64	8.091	1.068	0.3611	
3	-10	1.71	0.01	1.7	3	1	0.025	7.99	7.808	1.023	0.3245	
2	-10	1.66	0.01	1.65	3	1	0.025	7.673	7.667	1.001	0.307	
5	-10	1.633	0.01	1.623	3	1	0.025	7.501	7.59	0.9884	0.2977	

Sec	w	A_T	K_T	K_m	Q_m	S_f	h_L	α	V	KE	H	ΔH
			10^3	10^3		10^{-3}	10^{-3}					
1	1.40	6.187	0.2285	0.2285	20	7.659	0	1	3.232	0.5325	1.937	0
2	1.61	7.36	0.2901	0.2593	20	5.949	-59.49	1	2.717	0.3764	1.986	0.0104
2	1.81	8.64	0.3611	0.2948	20	4.603	-46.03	1	2.315	0.2731	2.083	-0.0998
2	1.71	7.99	0.3245	0.2765	20	5.231	-52.31	1	2.503	0.3194	2.029	-0.0398
2	1.66	7.673	0.307	0.2678	20	5.578	-55.78	1	2.607	0.3463	2.006	-0.0133
2	1.633	7.501	0.2977	0.2631	20	5.777	-57.77	1	2.666	0.3623	1.995	-3.e-10

Here H is the hydraulic head and ΔH is the difference in head, i.e. $\Delta H = H_u - H_d - h_l$.

Common Errors: wrong yc; using $y = 1.6$ at $x = 0$; missing z at sec. 2; using direct step method; using compound channel approach with alpha; averaging two iterated values;

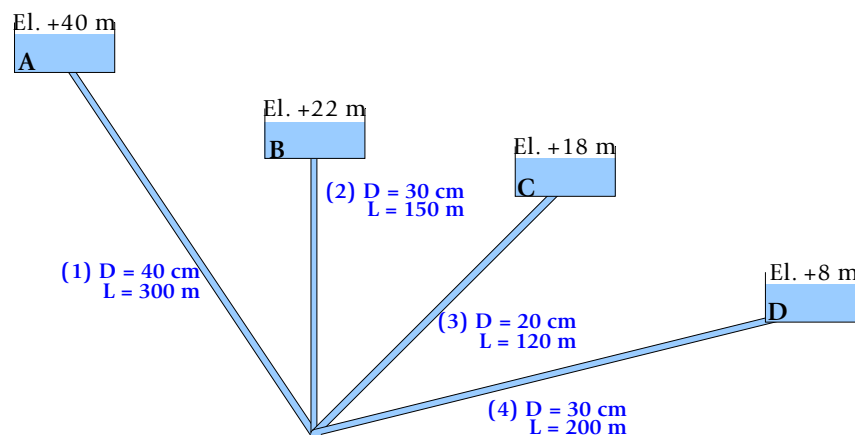
Homework Problems 2010S

HomeWork I

NAME:

ID #:

Determine the discharge in the pipes assuming $f = 0.02$. Neglect minor losses.



Solution

The head at the junction J is somewhere between 40 and 8 m. The first step is to determine the range more specifically, either between 40 and 22 m, or 22 and 18 m, or 18 and 8 m. Assuming $H_j = 22$ m, one obtains that $Q_1 = 0.61$ m³/s, $Q_3 = 0.08$ m³/s, and $Q_4 = 0.32$ m³/s.

The flow surplus between pipe 1 and pipes 3 & 4 implies that Reservoir A is supplying all other reservoirs. Hence, the head at the junction must be greater than 22 m to account for it. (Try $H_j = 18$ m and see the results)

The continuity equation at the junction is therefore

$$Q_1 = Q_2 + Q_3 + Q_4$$

Expressing the energy equation between the junction J and all four reservoirs, one obtains

$$\begin{aligned} 40 = H_j + \left(0.02 \frac{300}{0.4}\right) \frac{V_1^2}{2g}; & & H_j = 22 + \left(0.02 \frac{150}{0.3}\right) \frac{V_2^2}{2g} \\ H_j = 18 + \left(0.02 \frac{120}{0.2}\right) \frac{V_3^2}{2g}; & & H_j = 8 + \left(0.02 \frac{200}{0.3}\right) \frac{V_4^2}{2g} \end{aligned}$$

Substituting the velocity terms in the continuity equation ($Q = VA$) and solving for the unknown head at the junction H_j by iteration, one obtains $H_j = 23.91$ m. The flow rates are therefore $Q_1 = 0.5765$ m³/s, $Q_3 = 0.1368$ m³/s, $Q_3 = 0.098$ m³/s, and $Q_4 = 0.3420$ m³/s.

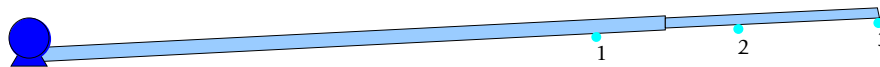
Common Errors: wrong deduction of flow directions, missing area term; insufficient iteration so that continuity is not conserved.

HomeWork II

NAME:

ID #:

Compute the minimum pump power required to supply an irrigation pipe system composed of a supply line and three port outlets laid on an uphill slope of 10% as shown. There are 3 discharge openings that are 2 m spaced apart and the distance from the pump to the first opening is 20 m. The diameter of the manifold pipe is 5 cm for the first 21 m and 3 cm for the last 3 m. The diameter of the ports is 1 cm and the discharge rate at the last port in the pipe manifold system is 0.2 l/s. Use a constant orifice discharge coefficient $C_D = 0.675$ for the ports, an equivalent sand-grain roughness $k_s = 0.12$ mm for the pipe, an expansion coefficient of 0.15, a contraction coefficient of 0.20, a kinematic viscosity of water of 1×10^{-6} m²/s, and a pump efficiency of 80%. Determine also the minimum pressure in the pipe system.



Solution

The flow rates in the manifold system is obtained from the continuity equation at each port $Q_i = Q_{i+1} + q_i$. The discharge q from the orifice (port) is obtained from

$$q_o = av_o = aC_D \sqrt{2g \left(\frac{p_i}{\gamma} + \frac{v_i^2}{2g} + z_i - \frac{p_o}{\gamma} + z_o \right)} = aC_D \sqrt{2g \left(h_i + \frac{v_i^2}{2g} - h_o \right)}$$

The head upstream is obtained by expressing the energy equation along the pipe segment

$$H_i = H_{i+1} + f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} \quad H = \frac{p}{\gamma} + \frac{v^2}{2g} + z$$

Here, there is a change in pipe diameter that affects the friction loss term and introduces a contraction loss term. The solution proceeds from the last port using the orifice discharge equation to get the head at the last outlet. The piezometric head outside is the elevation assumed to be equal to $0.1x$ rather than $\sin(\text{atan}(0.1))x$. The solution is as follows:

Port or Segment	x (m)	z (m)	q (m ³ /s)	Q (m ³ /s)	D (m)	V (m/s)	Re 10^3	f	h_1 (m)	H (m)
3	24	2.4	0.000200	0.000200	0.03	0.2829	84.88	0.0381	0.0104	3.1254
2s	22	2.2	0.000227	0.000427	0.03	0.6043	18.13	0.0339	0.0248	3.1358
2l	21	2.1		0.000427	0.05	0.2175	10.87	0.0342	0.0017	3.1605
1	20	2	0.000253	0.000680	0.05	0.3465	17.32	0.0315	0.0772	3.1622
Pump	0	0								3.2393

Therefore, the pump power required is $P = \gamma QH = 9.81(0.00068)3.239/0.8 = 27$ W. The minimum pressure is at port 3 and is obtained from

$$H_3 = 3.1254 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + 2.4 \quad \text{i.e.} \quad \frac{P_3}{\gamma} = 0.72 \text{ m}$$

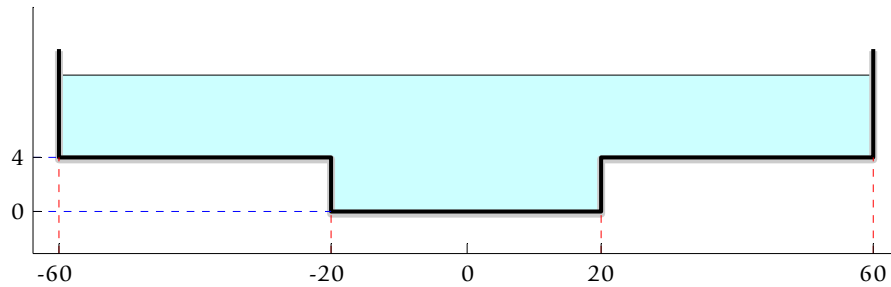
Common Errors: missing z in h_o ; missing contraction loss; missing dual friction terms;

HomeWork III

NAME:

ID #:

Compute the minimum height of the levees (side walls) necessary to protect from the 100-yr flood of magnitude of $300 \text{ m}^3/\text{s}$. The main channel is covered with gravel beds ($n=0.035$) while the overbank areas are covered with tall grass ($n=0.040$). The uniform slope of the channel is 0.25%.



Solution

From Manning's equation, the depth for a flow of $300 \text{ m}^3/\text{s}$ is 2.85 m. There is no need for levees!

HomeWork IV

NAME:

ID #:

A 2 m high broad crested weir is used to measure the flow in a rectangular open channel 4 m wide. The weir is 10 m long and 4 m wide. Compute the depth upstream and downstream from the weir for a channel discharge of $40 \text{ m}^3/\text{s}$. Assume $C_d = 0.95$.

Solution

Using the broad-crested weir equation, one obtains that $H = 3.36 \text{ m}$. Hence, the depth upstream is $y_u = 5.36 \text{ m}$

Using the energy equation between the critical section at the weir and a point downstream, one can find the depth downstream. The critical depth is $y_c = 2.17 \text{ m}$ and the depth downstream after few iterations is $y_d = 1.11 \text{ m}$.

HomeWork V

NAME:

ID #:

Water flows from under a sluice gate into a 1.5 m wide horizontal rectangular concrete channel at a rate of 3 m³/s per meter of width. The depth at a section is 20 cm. Compute the water surface profile up to a depth of 50 cm in depth increments of 10 cm. Manning's roughness $n = 0.013$.

Solution

The flow regime for a depth of 20 cm and 50 cm can be obtained from the Froude number

$$F_r = \frac{V}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}} = \frac{3}{0.2\sqrt{9.81(0.2)}} = 10.71$$

for the depth of 20 cm and $Fr = 2.71$ for the depth of 50 cm. Hence, the flow is supercritical throughout and the water surface profile is below the critical depth, i.e. H3. One can also reach the same conclusion by comparing the given depths with the computed critical depth $y_c = 0.97$ m.

The length of the reach Δx is solved using the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \quad E = y + \alpha \frac{V^2}{2g} \quad S_f = \frac{Q^2}{K^2} \quad K = \frac{1}{n} AR^{2/3}$$

Here $S_0 = 0$ and $n = 0.013$ for a horizontal concrete channel. The water surface profile is calculated using the direct step method since the channel configuration is uniform. The upstream depth is given as 0.2 m.

Section	Depth	A	P	R	K	V	E	K_m	S_f	Δx	x
1	0.2	0.3	1.9	0.1579	6.741	15	11.67	0	0	0	0
2	0.3	0.45	2.1	0.2143	12.4	10	5.397	9.569	221.2	28.35	28.35
3	0.4	0.6	2.3	0.2609	18.84	7.5	3.267	15.62	83	25.66	54.01
4	0.5	0.75	2.5	0.3	25.85	6	2.335	22.35	40.54	22.99	77

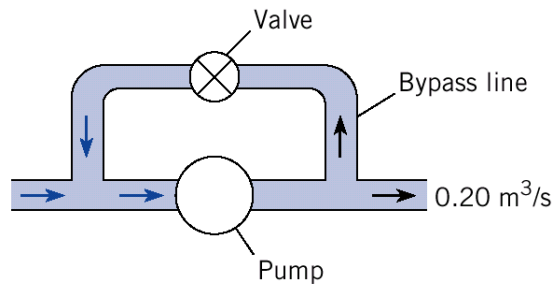
Homework Problems 2010F

HomeWork I

NAME: Bypass Problem

ID #:

Compute the discharge through the pump and the bypass line. The head-versus-discharge curve for the pump is given by $h_p = 100 - 100Q^2$ where h_p is in meters and Q is in m^3/s . The bypass line is 10 cm in diameter and the gate valve is wide open. Neglect the pipe friction.



Solution

The continuity equation gives $Q_p = Q_b + 0.2$ or $Q_1 = Q_2 + 0.2$.

The energy equation between node 1 and node 2 in the pump branch is

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

The energy equation between node 1 and node 2 in the valve branch is

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + 0.2 \frac{V_1^2}{2g}$$

Equating the two equations, one obtains

$$h_p = 0.2 \frac{V_1^2}{2g}$$

Or

$$100 - 100Q_1^2 = 0.2 \frac{Q_2^2}{2gA_2^2}$$

Substituting and solving

$$100 - 100(Q_2 + 0.2)^2 = 0.2 \frac{Q_2^2}{2gA_2^2}$$

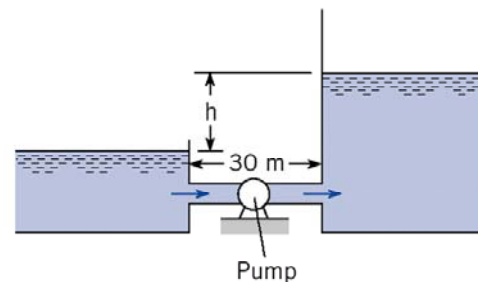
One obtains $Q_2 = 0.531 m^3/s$ and $Q_1 = 0.731 m^3/s$.

HomeWork II

NAME: Time-to-Fill

ID #:

The pump shown below is used to fill a tank from a reservoir. The head provided by the pump is given by $h_p = h_o \left(1 - Q^2/Q_{\max}^2\right)$ where h_o is 50 meters, Q is the discharge through the pump and Q_{\max} is $2 \text{ m}^3/\text{s}$. Assume that $f = 0.018$ and the pipe diameter is 90 cm. The cross-sectional area of the tank is 100 m^2 and the initial water level in the tank is the same as the level in the reservoir. Compute the time it will take to fill the tank to a height h of 40 m.



Solution

Expressing the energy equation between the two reservoirs, one obtain

$$0 + h_p = h + \left(0.5 + 0.018 \frac{30}{0.9} + 1\right) \frac{V^2}{2g}$$

Substituting the pump equation

$$50 \left(1 - \frac{Q^2}{4}\right) = h + \left(0.5 + 0.018 \frac{30}{0.9} + 1\right) \frac{Q^2}{2gA^2}$$

Expressing Q in terms of h and using $A = 0.636 \text{ m}^2$, one gets

$$50 - h = 12.5Q^2 + 1.05 \frac{Q^2}{gA^2} = 12.765Q^2$$

Writing the continuity equation in the tank

$$Q = A \frac{dh}{dt} = 100 \frac{dh}{dt}$$

Combining the two equations, one obtains

$$\frac{\sqrt{50-h}}{3.573} = 100 \frac{dh}{dt}$$

Integrating using the initial condition $h = 0$ at $t = 0$

$$t = 714.6 \left[7.071 - \sqrt{50-h}\right]$$

Therefore, the time it will take to fill the tank to 40 m is 2793 s or 46.5 min.

HomeWork III

NAME: Weir Height

ID #:

A rectangular irrigation canal 3 m wide carries water with a discharge of 6 m³/s. What height of rectangular weir installed across the canal will raise the water surface to a level 2 m above the canal floor? The discharge equation is given by $Q = K\sqrt{2g}LH^{3/2}$ with the flow coefficient K expressed by $K = 0.40 + 0.05H/P$ where P is the height of the weir.

Solution

The discharge equation is

$$Q = \left(0.4 + 0.05 \frac{H}{P}\right) \sqrt{2g}LH^{1.5}$$

The upstream water surface level is given as $P + H = 2$; hence, the discharge equation becomes

$$Q = \left(0.4 + 0.05 \frac{2-P}{P}\right) \sqrt{2g}L(2-P)^{1.5}$$

Substituting the values, one obtains

$$6 = \left(0.4 + 0.05 \frac{2-P}{P}\right) 3\sqrt{2(9.81)}(2-P)^{1.5}$$

Solving for P , one gets $P = 1$ m

Common Errors: confusion with $H-P=2$ or $H=2$; derivation of y_c at weir and EQ with upstream;

HomeWork IV

NAME:

ID #:

A hydraulic jump occurs in a rectangular channel that is 12 ft wide. If the depth upstream of the jump is 1 ft and the depth downstream of it is 8 ft, what is the rate of energy (power) loss in horsepower produced by the jump? (1 HP = 550 lb-ft/s)

Solution

The energy loss in a hydraulic jump in a rectangular channel is

$$h_l = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

where y_2 is the depth downstream of the hydraulic jump. Substituting the given depth values, the energy loss is $h_l = 10.72$ ft.

The power loss in horsepower is given by $P = \gamma Q h_l / 550$. The flow rate is obtained from the Froude number from the sequent depths relationship for rectangular channels

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8 F_{r1}^2} - 1 \right)$$

The Froude number is calculated to be equal to 6. The Froude number is defined by $F_{r1} = V_1 / \sqrt{g y_1}$ where V_1 is the velocity in the supercritical flow. Hence, $V_1 = 34$ ft/s and the flow rate is 408.5 ft³/s. The power loss is

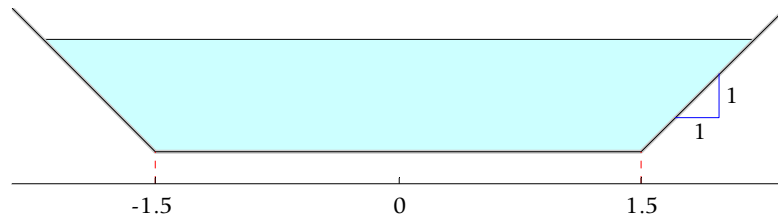
$$P = \frac{\gamma Q h_l}{550} = \frac{62.47(408.5)10.72}{550} = 497 \text{ HP}$$

HomeWork V

NAME:

ID #:

A trapezoidal channel with the dimensions as shown is laid on a slope of 0.001. The Manning's n value for this channel is 0.025 and the rate of flow is $20 \text{ m}^3/\text{s}$. Compute the water surface profile from the free outfall section to the point where the depth is 1.6 m using a single reach



Solution

The length of the reach Δx is obtained from the energy equation

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \quad E = y + \alpha \frac{V^2}{2g} \quad S_f = \frac{Q^2}{K^2} \quad K = \frac{1}{n} AR^{2/3}$$

The critical depth at the free outfall section is obtained from

$$\frac{TQ^2}{gA^3} = \frac{(20)^2 (b + 2my)}{9.81 (by + my^2)^3} = 1$$

Solving for y , one gets $y = 1.405 \text{ m}$. The water surface profile is calculated using the direct step method from 1.405 m to 1.6 m in one single reach.

Section	Depth	A	P	R	K	V	E	K_m	$S_f (10^3)$	Δx	x
1	1.405	6.189	6.974	0.8875	228.6	3.232	1.937	0	0	0	0
2	1.6	7.36	7.525	0.978	290.1	2.717	1.976	259.3	5.947	-7.906	-7.906

The results when the friction slope is given by $S_f = n^2 V^2 / R^{4/3}$ are

Section	y	R	V	E	Mean R	Mean V	S	Δx	x
							10^3		
1	1.405	0.8873	3.232	1.937	0	0	0	0	0
2	1.6	0.978	2.717	1.976	0.9327	2.975	6070	-7.714	-7.714

Common Errors: wrong yc; wrong R; missing S_0

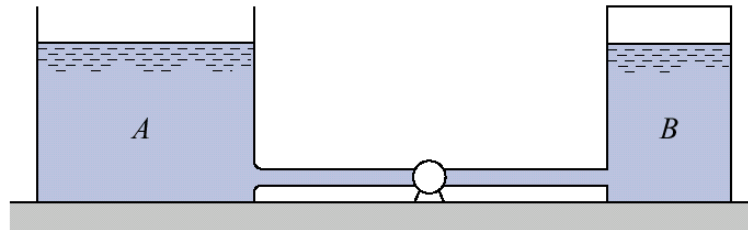
Homework Problems 2011

HomeWork I

NAME:

ID #:

Determine the power necessary to pump water from an open reservoir *A* to a pressurized tank *B* given that the pump efficiency is 90%. The initial water levels in the tanks are the same, the discharge is 0.03 m³/s, and the pressure in tank *B* is 70 kPa. The 10 cm diameter steel pipe ($k_s = 0.046$ mm) is 90 m long.



Solution

Expressing the energy equation between the water surfaces in the two reservoirs, one gets

$$0 + 0 + 0 + h_p = \frac{70}{\gamma} + 0 + 0 + \left(k_e + k_x + f \frac{L}{D} \right) \frac{V^2}{2g}$$

The discharge rate is given as 0.03 m³/s and the velocity in the pipe is then 3.82 m/s. The Reynolds number is 382 000 and the friction factor is given by

$$f = 0.25 \left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^{-2} = 0.0178$$

The minor loss coefficients are $k_e = 0.12$ for entrance and $k_x = 1$ for exit. Therefore, the sum of head losses is $(16 + 1.12) V^2 / 2g = 12.75$ m.

Hence, the pump head is $h_p = 19.9$ m and the pump horsepower for 90% efficiency is

$$P = \frac{\gamma Q h_p}{0.9} = 6.5 \text{ kW}$$

Results Summary

k_m	f	V (m/s)	KE (m)	h_1 (m)	h_p (m)	P (kW)
1.12	0.0178	3.82	0.7445	12.75	19.9	6.5

Common Errors: Forgotten minor losses; Inclusion of velocity head at B

HomeWork II

NAME:

ID #:

Calculate the water surface elevation needed in reservoir 1 to produce a discharge to the atmosphere of 20 l/s at the end of *Infinity Road* (junction 7). Pipes 1 through 6 are 20 cm in diameter while pipes 7 and 8 are 15 cm in diameter. The length of the pipes are 300 m (pipes 1, 3, & 8), 250 m (pipes 2 & 7), 400 m (pipes 4 & 5), 500 m (pipe 6). The pipes are concrete with a roughness coefficient $k_s = 0.5$ mm and all nodes are at elevation 550 m. Determine also the discharge to the atmosphere at nodes 8 and 9.

Solution

Expressing the energy equation between node 5 and 7, one gets

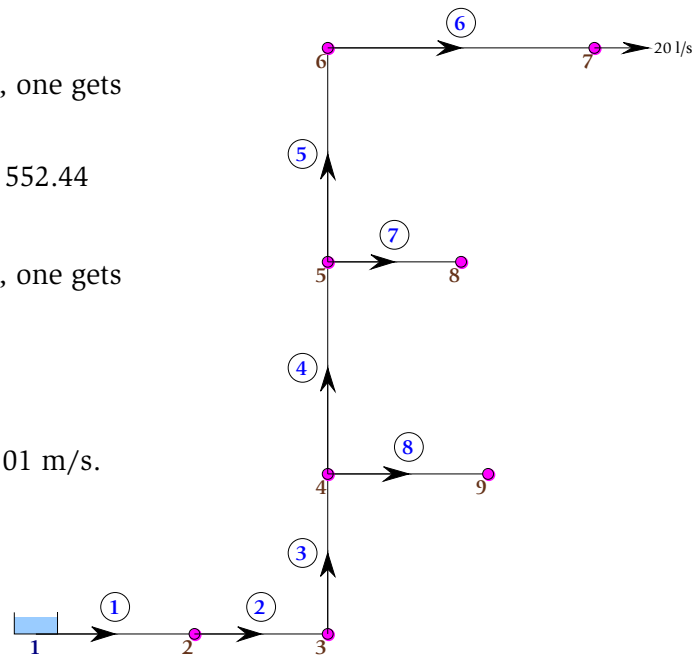
$$H_5 = H_7 + f \frac{L_{5,6}}{D_{5,6}} \frac{V_{5,6}^2}{2g} = 550 + \left(1 + 0.0266 \frac{900}{0.2} \right) \frac{0.637^2}{19.62} = 552.44$$

Expressing the energy equation between node 5 and 8, one gets

$$H_5 = H_8 + f \frac{L_7}{D_7} \frac{V_7^2}{2g} = 550 + \left(1 + f \frac{250}{0.15} \right) \frac{V_7^2}{2g} = 552.44$$

Iterating on V_7 and f , one obtains $f = 0.028$ and $V_7 = 1.01$ m/s.

Hence, $Q_7 = 0.0179$ m³/s.



The discharge in pipe 4 is now the sum of the two discharges, i.e. $Q = 0.0379$ m³/s.

Expressing the energy equation between node 4 and 5, one gets

$$H_4 = H_5 + f \frac{L_4}{D_4} \frac{V_4^2}{2g} = 552.44 + \left(0.0257 \frac{400}{0.2} \right) \frac{1.206^2}{19.62} = 556.25$$

Expressing the energy equation between node 4 and 9, one gets

$$H_4 = H_9 + f \frac{L_8}{D_8} \frac{V_8^2}{2g} = 550 + \left(1 + f \frac{300}{0.15} \right) \frac{V_8^2}{2g} = 556.25$$

Iterating on V_8 and f , one obtains $f = 0.0277$ and $V_8 = 1.49$ m/s. Hence, $Q_8 = 0.0263$ m³/s.

The total discharge is then 64.2 l/s. Expressing the energy equation between the reservoir and node 4 using $Q = 0.0642$ m³/s, one obtains

$$H_1 = H_4 + f \frac{L_{1,2,3}}{D_{1,2,3}} \frac{V_{1,2,3}^2}{2g} = 556.25 + \left(0.0254 \frac{850}{0.2} \right) \frac{2.044^2}{19.62} = 579.4$$

Common Errors: Neglected flow at nodes 8 and 9 in energy equation.

HomeWork III

NAME:

ID #:

Compute the depth of water for a discharge of $0.3 \text{ m}^3/\text{s}$ in a 1.2-m-diameter concrete culvert (drainage pipe). The slope of the barrel is 0.003 and the length is 20 m. Assume $n = 0.013$.

Solution

The discharge for a circular culvert is given by

$$Q_c = \frac{1}{n} A_c \left(\frac{A_c}{P_c} \right)^{2/3} \sqrt{S} \quad A_c = \frac{D^2}{8} (\theta - \sin \theta) \quad P_c = \frac{D\theta}{2}$$

The depth of water in a circular channel is given by

$$y = \frac{D}{2} \left(1 - \cos \frac{\theta}{2} \right)$$

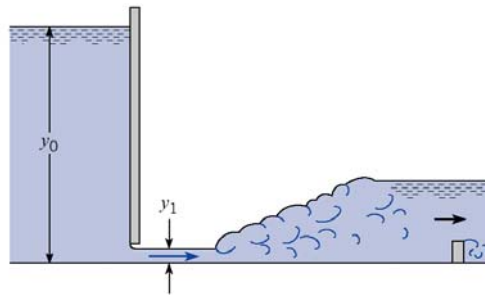
Solving Manning's equation for the lower arc angle θ with $Q = 0.3$, $D = 1.2$, $S = 0.003$, and $n = 0.013$, one obtains $\theta = 2.109$ radians (120°). Hence, the water depth is 303 mm.

HomeWork IV

NAME:

ID #:

Water is flowing under the sluice gate in a horizontal rectangular channel that is 5 ft wide. The depth y_0 and y_1 are 65 ft and 1 ft, respectively. Calculate the energy loss in the hydraulic jump in ft and the power loss in horsepower.



Solution

The energy loss in a hydraulic jump in a rectangular channel is

$$h_l = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

where y_2 is the depth downstream of the hydraulic jump. For rectangular channels, the sequent depths are related through

$$y_2 = \frac{y_1}{2} (\sqrt{1 + 8F_{r1}^2} - 1)$$

The Froude number is $F_{r1} = V_1 / \sqrt{gy_1}$ where V_1 is the velocity downstream of the gate. To compute V_1 , one writes the energy equation from a section upstream to a section immediately downstream of the gate. Assuming negligible loss under the gate, the energy equation is

$$65 = 1 + \frac{V_1^2}{2g}$$

Hence, $V_1 = \sqrt{64(64.4)} = 64.2$, $F_{r1} = 11.3$, and $y_2 = 15.5$ ft.

The energy loss is

$$h_l = \frac{(y_2 - y_1)^3}{4y_1y_2} = 49.2 \text{ ft.}$$

The power loss in horsepower is

$$P = \frac{\gamma Q h_l}{550} = \frac{62.4 (64.2) (1) (5) 49.2}{550} = 1793 \text{ HP}$$

Common Errors: Conversion to HP 550; deriving the momentum equation for sequent depth; using an "exit" or "entrance" loss term in energy equation; confusing power loss with energy loss.

American University of Beirut
Department of Civil & Environmental Engineering

CIVE 440 Hydraulics & Laboratory
Due Date: 14-Apr-11

Spring 2011
Origin: Quiz 501

HomeWork V

NAME:

ID #:

Water flows at $600 \text{ ft}^3/\text{s}$ in a rectangular channel 22 ft wide with $n = 0.024$ and a slope of 0.0015. A dam across the channel increases the depth to 15 ft. Estimate the distance L upstream at which the water depth will be 12 ft using the direct step method with 1 ft depth increment. What is the water depth asymptotically far upstream?

Solution

The solution was stolen! Please help me find it.

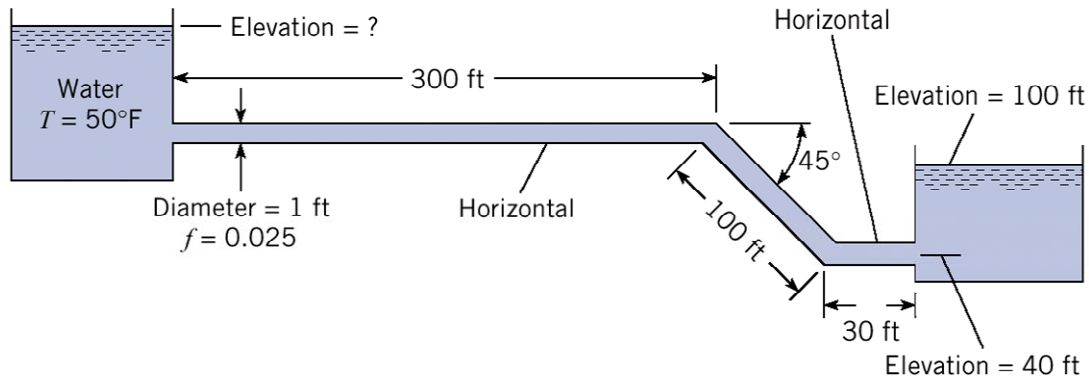
Homework Problems 2011F

HomeWork I

NAME:

ID #:

Estimate the elevation required in the upper reservoir to produce a water discharge of 10 cfs in the system. Calculate also the minimum pressure in the pipe. The loss coefficient of the 45° elbow is 0.4.



Solution

Expressing the energy equation between the two reservoirs, one obtains

$$0 + 0 + z_1 = 0 + 0 + z_2 + \left(k_e + 2k_b + k_x + f \frac{L}{D} \right) \frac{V^2}{2g}$$

Substituting the values, one obtains

$$z_1 = 100 + \left(0.5 + 2(0.4) + 1 + 0.025 \frac{430}{1} \right) \frac{10^2}{(\pi/4)^2 2(32.2)} = 132.85 \text{ ft}$$

The minimum pressure occurs at a highest point in the system with a maximum head loss, which is located here downstream of the first bend. Expressing the energy equation between the upper reservoir and the first bend, one obtains

$$z_1 = \frac{p_b}{\gamma} + \frac{V^2}{2g} + z_b + \left(k_e + k_b + f \frac{L}{D} \right) \frac{V^2}{2g}$$

Substituting the values noting that $z_b = 40 + 70.71$ ft, one obtains

$$132.85 = \frac{p_b}{\gamma} + \frac{12.73^2}{64.4} + 110.71 + \left(0.5 + 0.4 + 0.025 \frac{300}{1} \right) \frac{12.73^2}{64.4}$$

Hence, the pressure head $p_b/\gamma = -1.5$ ft and the pressure is $p_b = -97$ lb/ft².

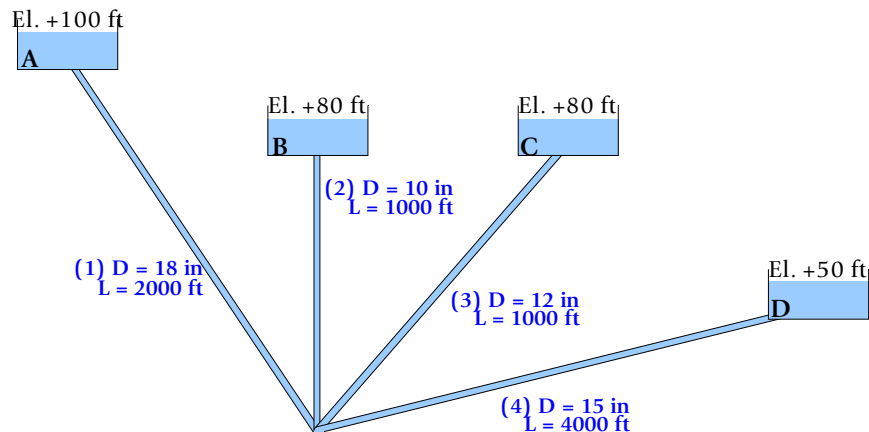
Common Errors: missing 1 or more minor loss terms (e.g. bend #2); incorrect pressure location; missing hl in pressure calculation; missing z of pipe;

HomeWork II

NAME:

ID #:

Determine the discharge in the pipes assuming $f = 0.02$. Neglect minor losses.



Solution

Expressing the energy equation between reservoir A and the junction, one obtains

$$100 = H_j + \left(0.02 \frac{2000}{1.5} \right) \frac{V_1^2}{2g} = H_j + 0.133Q_1^2$$

Expressing the energy equation between the junction and reservoir D, one obtains

$$H_j = 50 + \left(0.02 \frac{4000}{1.25} \right) \frac{V_4^2}{2g} = 50 + 0.66Q_4^2$$

Assuming that $H_j = 80$ ft, one obtains $V_1 = 6.95$ ft/s and $V_4 = 5.49$ ft/s. The flow rates are then $Q_1 = 12.28$ ft³/s and $Q_4 = 6.74$ ft³/s. The flow surplus between pipe 1 and pipe 4 implies that Reservoir A is supplying Reservoirs B and C. Hence, the head at the junction must be greater than 80 ft to account for it. Expressing the energy equation between the junction J and Reservoir B and Reservoir C, respectively, one obtains two additional equations

$$H_j = 80 + \left(0.02 \frac{1000}{10/12} \right) \frac{V_2^2}{2g} = 80 + 1.25Q_2^2; \quad H_j = 80 + \left(0.02 \frac{1000}{1} \right) \frac{V_3^2}{2g} = 80 + 0.5Q_3^2$$

The continuity equation at the junction is $Q_1 = Q_2 + Q_3 + Q_4$. Substituting the velocity terms in the continuity equation ($Q = VA$)

$$2.746\sqrt{100 - H_j} - 0.893\sqrt{H_j - 80} - 1.41\sqrt{H_j - 80} - 1.23\sqrt{H_j - 50} = 0$$

Solving for the unknown head at the junction H_j by iteration, one obtains $H_j = 83.24$ m. The flow rates are therefore $Q_1 = 11.25$ ft³/s, $Q_2 = 1.61$ ft³/s, $Q_3 = 2.54$ ft³/s and $Q_4 = 7.1$ ft³/s.

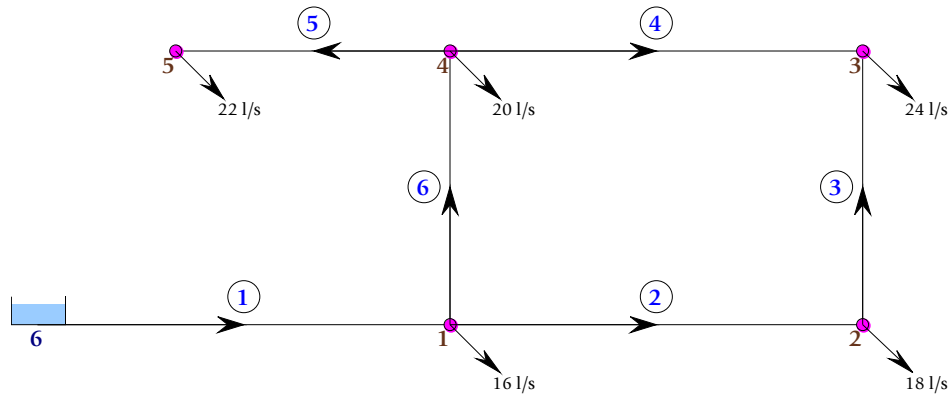
Common Errors: wrong deduction of flow directions, missing area term; insufficient iteration so that continuity is not conserved.

HomeWork III

NAME:

ID #:

Compute the flow rates in the network system assuming $f = 0.02$. The hydraulic data are tabulated below



Junction	1	2	3	4	5	6	Pipe	1	2	3	4	5	6
Elevation (m)	55	55	55	55	44	100	Length (m)	1500	300	200	300	500	200
Demand (l/s)	16	18	24	20	22		Diameter (cm)	30	20	20	20	20	20

Solution

There are 6 unknown pipe flow rates and 6 equations (5 continuity equations and the energy equation around the loop). The equations are

$$\text{J-1: } Q_1 = Q_2 + Q_6 + 0.016$$

$$\text{J-2: } Q_2 = Q_3 + 0.018$$

$$\text{J-3: } Q_4 + Q_3 = 0.024$$

$$\text{J-4: } Q_6 = Q_4 + Q_5 + 0.02$$

$$\text{J-5: } Q_5 = 0.022$$

$$\text{Loop 2346: } k_2 |Q_2| Q_2 + k_3 |Q_3| Q_3 - k_4 |Q_4| Q_4 - k_6 |Q_6| Q_6 = 0$$

Here $k = 8f / g\pi^2 \cdot L / D^5$

Pipe	1	2	3	4	5	6
$k \text{ (s}^2/\text{m}^5) \times 10^3$	1.0201	1.5493	1.0328	1.5493	2.5821	1.0328

Expressing the loop equation in terms of Q_4 using the continuity equations, one gets

$$k_2 |0.018 + 0.024 - Q_4| (0.018 + 0.024 - Q_4) + k_3 |0.024 - Q_4| (0.024 - Q_4) - k_4 |Q_4| Q_4 - k_6 |Q_4 + 0.022 + 0.02| (Q_4 + 0.022 + 0.02) = 0$$

For $Q_4 = 1 \text{ l/s}$ ($0.001 \text{ m}^3/\text{s}$), the equation gives 1.24 (rather than zero), while for $Q_4 = 10 \text{ l/s}$, the above equation gives -1.16 . Iterating further on Q_4 , one obtains $Q_4 = 5.65 \text{ l/s}$.

From the junction equations, the flow rates are: $Q_3 = 18.35 \text{ l/s}$, $Q_2 = 36.35 \text{ l/s}$, and $Q_6 = 47.65 \text{ l/s}$. $Q_1 = 100 \text{ l/s}$ is the sum of all demands as expected.

Common Errors: inconsistent units; sign in loop equation; incorrect algebraic manipulation.

HomeWork IV

NAME:

ID #:

Water flows uniformly in a 2-m-wide rectangular channel at a rate of $1.6 \text{ m}^3/\text{s}$ and a depth of 0.75 m. Determine the maximum width or minimum contraction required to cause critical depth at the contraction.

Solution

- The depth at section 1 is greater than the critical depth (0.403 m) and the flow is thence subcritical.
- Expressing the energy equation between section 1 and 2 at the constriction, one gets

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g}$$

- Here $z_1 = z_2 = 0$, $y_1 = 0.75$, and $V_1 = 1.067 \text{ m/s}$.
- The velocity at section 2 is given by $V_2 = 1.6 / (by_2)$ where y_2 is the critical depth
- The critical depth is given by $y_c = (q^2/g)^{1/3}$ where $q = Q/b$
- Substituting in the energy equation and solving for b , one obtains $b = 1.29 \text{ m}$
-
- One can also use the property that at a critical section $V_c = \sqrt{gy_c}$. Therefore

$$y_2 + \frac{V_2^2}{2g} = y_c + \frac{gy_c}{2g} = \frac{3}{2}y_c$$

- Hence

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} = z_2 + \frac{3}{2}y_c$$

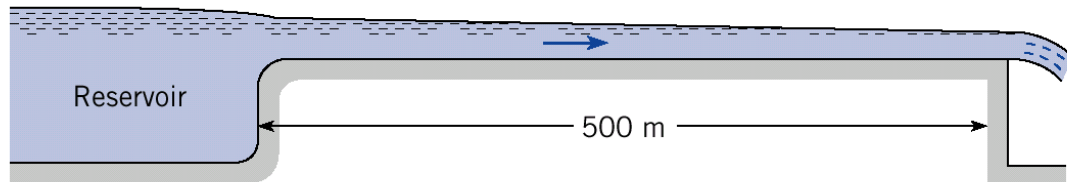
- $y_2 = y_c = 0.539 \text{ m}$ and $b = Q/\sqrt{gy_c^3} = 1.29 \text{ m}$

HomeWork V

NAME:

ID #:

Calculate the discharge in the steep rectangular concrete spillway. The channel is 4 m wide and 500 m long. The water surface elevation in the reservoir is 2 m above the channel bottom. Neglect the head loss at the entrance.



Solution

For a steep channel, the flow is supercritical. Therefore, the depth at the entrance is critical. Expressing the energy equation between the reservoir and the critical section, one gets

$$2 = y_2 + \frac{V_2^2}{2g}$$

Using the critical depth equation, one gets

$$y_2 = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{B^2 g}\right)^{1/3}$$

Substituting in the energy equation and solving for Q , one obtains $Q = 19.2 \text{ m}^3/\text{s}$.

One can also write the energy equation at the critical section using $V_c = \sqrt{gy_c}$

$$2 = y_2 + \frac{V_2^2}{2g} = y_c + \frac{gy_c}{2g} = \frac{3}{2} y_c$$

The critical depth is then $y_c = 4/3$, the velocity is $V_c = \sqrt{gy_c} = 3.62 \text{ m/s}$, and the flow is $Q = 19.2 \text{ m}^3/\text{s}$.

A third and quicker approach is to realize that the spillway is like a broad-crested weir. Hence

$$Q = 0.385L\sqrt{2g}H^{1.5} = 0.385(4)\sqrt{2(9.81)}2^{1.5} = 19.3 \text{ m}^3/\text{s}$$

Note: One can check for steepness by comparing with the normal depth if the value of the bottom slope was available.

Common Errors: critical depth at outfall; critical depth = 2; Velocity in reservoir using $y = 2$; $L = 500$

Experiments

Experiment #1: Fluid Friction & Minor Losses in Pipes

The purpose of this experiment consists of applying the energy equation to determine the friction factor f of pipes as a function of the Reynolds number and to find the minor loss coefficients of pipe fittings.

Experimental Procedure

- Turn on the pump on the hydraulic bench and adjust the flow control for an initial flow.
- Turn on the PC and run the software for the equipment.
- Open the valve for the segment pipe under consideration and close all the other valves.
- Connect 2 arbitrary sensors to the nodes enclosing the segment under study. Note their respective alphanumeric ID.
- Enable the reception of data from the chosen sensors in the computer program by clicking on their ID. Disable all others.
- Run the program and record the head measurements at each node as displayed in the program interface.
- Stop the program.
- Without changing the flow, repeat the above procedure on the expansion or contraction piece and on the 90° bend measurement outlets.
- Repeat the experiment for 3 to 4 different flow rates and make sure to reset or stop between each run.
- Determine the dimensions of the segment under study and record all necessary geometric and hydraulic data as tabulated in the accompanying graph on the equipment board.
- The experiment should be carried out for the straight pipe segment (friction loss) as well as the contraction and expansion fittings (minor loss).

Analysis and Discussion

- Computations and plots
 - Use the energy equation to find the friction factor of the pipe using the Darcy-Weisbach formula. Calculate the Reynolds number for each reading.
 - Use the energy equation to find the minor loss coefficient for each fitting. Note that you need to determine the friction factor for your calculations.
- Comparisons and discussions
 - Compare your f values with the values shown in the Moody diagram or tables and comment on the results.
 - Compare with the values given in a textbook and justify the sources of errors.

References

- Crowe, C. T., Roberson, J. A., and Elger, *Engineering Fluid Mechanics*, 7th ed., sec. 10.4-10.5, John Wiley & Sons, New York, 2001.
- Daugherty, R. L., J. B. Franzini, and E. J. Finnemore, *Fluid Mechanics with Engineering Applications*, 8th ed., pp. 203-212 & 223-240, McGraw-Hill, Singapore, 1989.

Laboratory Data Sheet

Friction Loss Trials

Reading	Flow	h_u	h_d
1			
2			
3			
4			
	Length	Diameter	Roughness
			Other Data

Expansion or Contraction Loss Trials

Reading	Flow	h_u	h_d
1			
2			
3			
4			
	Diameter 1	Diameter 2	Roughness
			Other Data

Bend Trials

Reading	Flow	h_u	h_d
1			
2			
3			
4			
	Diameter 1	Diameter 2	Roughness
			Other Data

Comments:

Group No: _____

Data Keeper: _____

Date: _____

Laboratory Data Calculations

Compute the friction factor f and the minor loss coefficients k_e (expansion or contraction) and k_b (bend) for one set of reading and compare with textbook values.

Summary of Results

	Measured	Textbook	Error	Measured	Textbook	Error	Measured	Textbook	Error
Q	f	f	%	k_e	k_e	%	k_c	k_c	%

Group No: _____

Name: _____

Date: _____

Experiment #2: Sharp-Crested Weirs

The purpose of this experiment consists of determining the coefficient of discharge of various sharp-crested weirs.

Experimental Procedure

Mount the rectangular weir. Turn on the pump and allow water to flow freely. Shut off the supply and establish the zero level at which no water flows over the weir. Record the zero reading. Open the supply, measure the discharge and the head above the weir. Repeat for three different flow rates making sure that equilibrium conditions are prevailing in each measurement. Repeat the experiment for the triangular and trapezoidal weir.

Analysis and Discussion

- Derivations
 - Derive the weir discharge equation neglecting the velocity of approach and determine the coefficient of discharge C_d .
 - Derive the weir discharge equation without neglecting the velocity of approach for the rectangular weir only and compare.
- Computations and plots
 - Compute and plot C_d vs H
 - Plot Q vs H and determine K and n in the following equation: $Q = KH^n$.
- Comparisons and discussions
 - Compare your values of C_d with the values in the textbook.
 - Compare your values of n and K with the values in the textbook.
 - Justify all sources of errors and deduct a meaningful conclusion.

References

- Crowe, C. T., Roberson, J. A., and Elger, *Engineering Fluid Mechanics*, 7th ed., p. 607-611, John Wiley & Sons, New York, 2001.
- Roberson, J. A., and C. T. Crowe, *Engineering Fluid Mechanics*, 5th ed., p. 620-624, John Wiley & Sons, New York, 1995.
- Daugherty, R. L., J. B. Franzini, and E. J. Finnemore, *Fluid Mechanics with Engineering Applications*, 8th ed., p. 431-433, McGraw-Hill, Singapore, 1989.
- Roberson, J. A., Cassidy, J. J., and M. H. Chaudhry, *Hydraulic Engineering*, Houghton Mifflin Co., Boston, MA, 1988.
- French, R., *Open-Channel Hydraulics*, McGraw-Hill, New York, 1985.

Laboratory Data Sheet

Rectangular Weir

Dimensions of the weir:
H Zero level reading:

Reading	<i>V</i>	<i>t</i>	<i>H</i>

Triangular Weir

Dimensions of the weir:
H Zero level reading:

Reading	<i>V</i>	<i>t</i>	<i>H</i>

Trapezoidal Weir

Dimensions of the weir:
H Zero level reading:

Reading	<i>V</i>	<i>t</i>	<i>H</i>

Any comments:

Group No: _____

Data Keeper: _____

Date: _____

Laboratory Data Calculations

Compute the coefficient of discharge C_d for the rectangular, triangular, and trapezoidal weir using one reading of data for each weir and neglecting the velocity of approach.

Summary of Results

Weir	Measured Q	Theoretical Q	C_d
Rectangular			
Triangular			
Trapezoidal			

Group No: _____

Name: _____

Date: _____

Experiment #3: Ogee Spillway & Hydraulic Jump

The purpose of this experiment consists of

1. Computing the coefficient of discharge of the Ogee Spillway and
2. Studying the properties of the hydraulic jump

Experimental Procedure

Set the channel to a horizontal slope. Position the Ogee spillway. Turn on the pump. Establish a stationary hydraulic jump in the flume. Measure the discharge, the critical depth y_c at the spillway crest, the water depths upstream and downstream of the jump, and the depth at few points upstream and downstream of the spillway. Repeat for three different flow rates. Record the channel slope. The flume is $\frac{1}{2}$ ft wide.

Analysis and Discussion

- Derivations
 - Derive the spillway discharge equation and determine the coefficient of discharge C_d .
 - Derive the equation for the hydraulic jump in a rectangular channel using the momentum equation.
- Computations and plots
 - Compute and plot C_d vs H
 - Plot Q vs H and determine K and n in the following equation: $Q = KL\sqrt{2g}H^n$.
 - Compute the critical depth.
 - Plot the flow profile & show on it the various depths including y_c .
 - Determine the head loss in the hydraulic jump
 - Calculate the proportional constant C in the equation of the length of the hydraulic jump $L_j = Cy_2$ where y_2 is the depth downstream.
- Comparisons and discussions
 - Compare your values of C_d with the values in the textbook.
 - Compare your values of n and C with the values in the textbook.
 - Compare the experimental values of the depth downstream of the spillway with the predicted value from the energy equation (alternate depth).
 - Compare the experimental values of the depth downstream of the hydraulic jump with the theoretical value (sequent depth).
 - Justify the difference between measured and theoretical values by analyzing the sources of errors
 - Deduct a meaningful conclusion.

References

- Crowe, C. T., Roberson, J. A., and Elger, *Engineering Fluid Mechanics*, 7th ed., sec. 15.2, John Wiley & Sons, New York, 2001.
- Daugherty, R. L., J. B. Franzini, and E. J. Finnemore, *Fluid Mechanics with Engineering Applications*, 8th ed., pp. 354-358, 433-436 and sec. 11.19, McGraw-Hill, Singapore, 1989.
- Roberson, J. A., Cassidy, J. J., and M. H. Chaudhry, *Hydraulic Engineering*, Houghton Mifflin Co., Boston, MA, 1988.
- French, R., *Open-Channel Hydraulics*, McGraw-Hill, New York, 1985

Laboratory Data Sheet

Water Level	Upstream		Spillway		Downstream		Flow
Reading	1	2	3	4	5	6	Q
1							
2							
3							
Distance							
Spillway Height							

Any comments:

Group No: _____

Data Keeper: _____

Date: _____

Laboratory Data Calculations

For one set of readings:

1. Compute the spillway's coefficient of discharge C_d .
2. Compute the critical depth y_c and compare with measured values.
3. Compute the depth downstream of the spillway y_d and the depth downstream of the hydraulic jump y_2 , and compare with measured values.

Summary of Results

Discharge Coefficient			Critical Section			Downstream Spillway			Downstream of Jump		
Meas.	Comp.		Meas.	Comp.	Diff.	Meas.	Comp.	Diff.	Meas.	Comp.	Diff.
Q	Q	C_d	y_c	y_c	%	y_d	y_d	%	y_2	y_2	%

Group No: _____

Name: _____

Date: _____

Formulae Sheet I

$$E = mc^2$$

$$\sum \vec{F} = \frac{d(m\vec{V})}{dt}$$

$$Q_{in} - Q_{out} = \frac{dS}{dt}$$

$$R_e = \frac{\rho VD}{\mu}$$

$$F_r = \frac{V}{\sqrt{gh}}$$

$$M_a = \frac{V}{c}$$

$$C_p = \frac{\Delta p}{\rho V^2/2}$$

$$p = \gamma h$$

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

$$Q = VA$$

$$Q = \int_A V dA$$

$$\sum \vec{F} = \rho Q (\vec{V}_2 - \vec{V}_1)$$

$$h_f = \frac{32\mu LV}{\gamma D^2}$$

$$h_l = f \frac{L}{D} \frac{V^2}{2g}$$

$$S_f = \frac{n^2 V^2}{2.22 R^{4/3}}$$

$$\frac{dp}{dz} = -\gamma$$

$$v = \frac{\mu}{\rho}$$

$$P = T \cdot \omega$$

$$\frac{1}{\sqrt{f}} = 2 \log \frac{D}{k_s} + 1.14$$

$$\frac{1}{\sqrt{f}} = 2 \log(R_e \sqrt{f}) - 0.8$$

$$h_L = \frac{f}{4} \frac{L}{R_h} \frac{V^2}{2g}$$

$$R_h = \frac{A}{P}$$

$$h_l = \frac{K_E}{2g} (V_1^2 - V_2^2)$$

$$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

$$h_m = k_m \frac{V^2}{2g}$$

$$S_f = \frac{n^2 V^2}{2.22 R^{4/3}}$$

$$S_f = \frac{f V^2}{8gR}$$

$$K_i = 0.675 \sqrt{1 - \frac{V_{i+1}^2}{2g(H_i - H_0)}}$$

$$Q = \frac{C_D A_0 \sqrt{2g\Delta h}}{\sqrt{1 - (C_c^2 A_0^2 / A_1^2)}}$$

$$Q = \frac{C_D A_2 \sqrt{2g\Delta h}}{\sqrt{1 - (A_2^2 / A_1^2)}}$$

$$P = \gamma QH$$

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$V = C \sqrt{RS_0}$$

$$V = 1.318 C_h R_h^{0.63} S^{0.54}$$

$$h_f = 3.02 L D^{-1.167} (V/C_h)^{1.85}$$

$$\bar{V} = \frac{Q}{A}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_w = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s}{3.7D} + \frac{2.51}{R_e \sqrt{f}} \right)$$

$$q_i = K_i a_i \sqrt{2g(H_i - H_0)}$$

$$H_i = H_{i+1} + f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}$$

$$Q_i = Q_{i+1} + q_i$$

$$R_e \sqrt{f} = \frac{D^{3/2}}{v} \sqrt{\frac{2gh_f}{L}}$$

$$f = \frac{1}{4} \left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^{-2}$$

$$h_m = k_m \frac{V^2}{2g}$$

$$F = \gamma h_c A$$

$$\sum F_x = \rho_2 Q_2 V_{2x} - \rho_1 Q_1 V_{1x}$$

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{Cte}$$

Formulae Sheet II

$$E = mc^2$$

$$\sum F_x = \sum V_x \rho V \cdot A$$

$$Q_{\text{in}} - Q_{\text{out}} = \frac{dS}{dt}$$

$$R_e = \frac{\rho V D}{\mu}$$

$$F_r = \frac{V}{\sqrt{gh}}$$

$$Q = \frac{1}{n} AR_h^{2/3} S^{1/2}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_m = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$\bar{V} = \frac{Q}{A}$$

$$h_L = \frac{f}{4} \frac{L}{R_h} \frac{V^2}{2g}$$

$$Q = B_T \sqrt{g} \left(\frac{2}{3} \right)^{3/2} \left(H_a + \frac{V_a^2}{2g} \right)^{3/2}$$

$$Q = \frac{2}{3} C_d \sqrt{2g} LH^{3/2}$$

$$Q = 0.385 C_d \sqrt{2g} LH^{3/2}$$

$$f_m = \frac{q^2}{yg} + \frac{y^2}{2}$$

$$R_h = \frac{A}{P}$$

$$V = C \sqrt{RS_0}$$

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2) H^{5/2}$$

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0}$$

$$A = \frac{D^2}{8} (\theta - \sin \theta)$$

$$P = B + 2y\sqrt{1+m^2}$$

$$Q = \frac{C_D A_0 \sqrt{2g\Delta h}}{\sqrt{1 - (C_c^2 A_0^2 / A_1^2)}}$$

$$h_m = k_m \frac{V^2}{2g}$$

$$V = 1.318 C_h R_h^{0.63} S^{0.54}$$

$$h_L = \frac{(y_2 - y_1)^3}{4y_1 y_2}$$

$$Q = KH^{5/2}$$

$$S_f = \frac{Q^2}{K^2} \quad K = \frac{AR^{2/3}}{n}$$

$$S_f = \frac{n^2 V^2}{2.22R^{4/3}}$$

$$S_f = \frac{fV^2}{8gR}$$

$$\alpha = \sum_{i=1}^N \left(\frac{Q_i^3}{A_i^2} \right) \left(\sum_{i=1}^N A_i \right)^2 \left(\sum_{i=1}^N Q_i \right)^{-3}$$

$$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

$$y = \frac{D}{2} [1 - \cos(\theta/2)]$$

$$y_1 + \frac{\alpha_1 V_1^2}{2g} + S_0 \Delta x = y_2 + \frac{\alpha_2 V_2^2}{2g} + S_f \Delta x + C_l \left(\frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right)$$

$$E = y + \frac{V^2}{2g}$$

$$Q = C_w L \left(H + \frac{V_o^2}{2g} \right)^{3/2}$$

$$Q = C_D A \sqrt{2g\Delta h}$$

$$P = \gamma QH$$

$$y_1 + \frac{V_1^2}{2g} + z_1 = y_2 + \frac{V_2^2}{2g} + z_2 + S_f \Delta x$$

$$\frac{Q^2 T_c}{g A_c^3} = 1$$

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8F_{r1}^2} - 1 \right)$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$A = y(b + my) \quad m = \cot \theta$$

$$\alpha = \frac{A_T^2}{K_T^3} \sum_{i=1}^N \left(\frac{K_i^3}{A_i^2} \right)$$

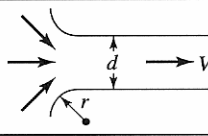
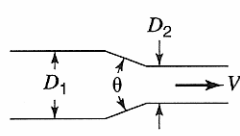
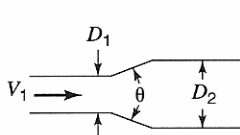
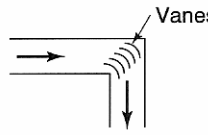
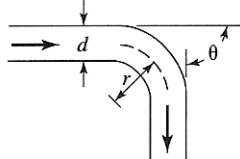
$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

$$h_f = 3.02 LD^{-1.167} (V/C_h)^{1.85}$$

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

Tables

Table 1. Minor loss coefficients for pipes

Description	Sketch	Additional Data	K	Source
Pipe entrance $h_{lm} = K_e V^2/2g$		r/d	K_e	(a)
		0.0	0.50	
		0.1	0.12	
		>0.2	0.03	
Contraction $h_{lm} = K_C V_2^2/2g$		D_2/D_1	K_C	(a)
			$\theta = 60^\circ$	K_C
		0.0	0.08	$\theta = 180^\circ$
		0.20	0.08	0.50
		0.40	0.07	0.49
		0.60	0.06	0.42
		0.80	0.05	0.32
0.90	0.04	0.18		
Expansion $h_{lm} = K_E V_1^2/2g$		D_1/D_2	K_E	(a)
			$\theta = 10^\circ$	K_E
		0.0	1.00	$\theta = 180^\circ$
		0.20	0.13	0.92
		0.40	0.11	0.72
90° miter bend		Without vanes	$K_b = 1.1$	(b)
		With vanes	$K_b = 0.2$	(b)
Smooth bend		r/d	K_b	(c)
			$\theta = 45^\circ$	K_b
		1	0.10	$\theta = 90^\circ$
		2	0.09	0.35
		4	0.10	0.19
6	0.12	0.16		
Threaded pipe fittings		Globe valve—wide open	$K_v = 10.0$	(b)
		Angle valve—wide open	$K_v = 5.0$	
		Gate valve—wide open	$K_v = 0.2$	
		Gate valve—half open	$K_v = 5.6$	
		Return bend	$K_b = 2.2$	
		Tee	$K_t = 1.8$	
90° elbow	$K_b = 0.9$			
45° elbow	$K_b = 0.4$			

(a) ASHRAE (1977)

(b) Streeter (1961)

(c) Bei (1938)

(d) Idel'chik (1966)

Source: after Roberson et al. (1988).

Table 2. Values of absolute roughness k_s for new commercial pipes

Pipe Material	k_s (mm)
Glass, plastic (smooth)	0.0
Copper or brass tubing	0.0015
Wrought iron, steel	0.046
Asphalted cast iron	0.12
Galvanized iron	0.15
Cast iron (average)	0.25
Concrete	0.3–3
Riveted steel	0.9–9
Rubber pipe (straight)	0.025

Table 3. Loss coefficients for channel transitions with subcritical flow

Transition Type	Contracting	Expanding
Abrupt	0.4-0.5	0.75-1.00
Cylinder-quadrant	0.2	0.5
Wedge	0.1-0.2	0.3-0.5
Warped	0.1	0.3

Table 4. Contraction and expansion angle for channel transitions

Transition Type	Contraction Angle	Expansion Angle
Abrupt	90°	90°
Wedge	27.5°	22.5°
Warped	12.5°	12.5°

Table 5. Values of manning's roughness n for rivers and channels

Bed Description	n
Concrete	0.013–0.015
Asphalt	0.016–0.018
Bare soil	0.020–0.023
Soil cement	0.020–0.025
Riprap	0.028–0.040
Rock cut	0.025–0.045

Table 6. Steepest recommended side slopes for channels

Material	side slope m (run to rise ratio)
Rock	0 – ¼
Earth with concrete lining	½
Stiff clay	1
Soft clay	1½
Loose sandy soil	2
Light sand, sandy loam	3

Table 7. Maximum permissible channel velocities

Material	V (m/s)
Fine sand	0.6
Coarse sand	1.2
Fine gravel	1.8
Sandy silt	0.6
Silt clay	1.0
Clay	1.8
Sedimentary rock	3.0
Soft sandstone	2.4
Soft shale	1.0
Igneous or hard rock	6.0

Ordinary human beings take everything either as a blessing or as a curse.
The uncommon ones take everything as a challenge.